Computational modelling of frost heave induced soil–pipeline interaction
II. Modelling of experiments at the Caen test facility

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Received 27 April 1998; accepted 29 June 1999

Abstract

This paper examines the three-dimensional computational modelling of the interaction between a buried pipeline and a soil region which is induced by effects of differential frost heave. The frost heave generation is modelled by the coupled processes of heat conduction and moisture transport within the soil mass and the mechanical processes in the soil accounts for elastic–viscoplastic phenomena. The pipeline is modelled as a circular structural element which possesses flexural, axial and shear stiffness characteristics. The computational procedures are used to develop estimates for the pipeline behaviour observed in the large-scale experimental facility at Caen, France, which investigated the behaviour of a buried pipeline located at a discontinuous frost heave zone. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Frost heave; Soil–pipeline interaction; Computational modelling; Experiments at Caen

1. Introduction

Buried pipelines provide an efficient and safe mode for the transport of energy resources such as oil, natural gas, coal slurries and for the transport of other materials such as mine tailings, hazardous waste and water. Pipelines which carry flammable energy resources in particular are designed, constructed and maintained according to standards and codes that safeguard their operation during their service life. The structural analysis and design of a buried pipeline should take into consideration the mutual interaction between the pipeline and the surrounding soil. These interactions can be induced by a variety of effects including service loads such as deformations due to temperature and internal pressure, loadings of a geotechnical nature including ground subsidence, frost heave, thaw settlement, ground swelling in expansive soils, action of external loadings

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PII: S0165-232X(99)00029-4
including berm construction, roadway traffic, earthquake loads, landslides (both onshore and offshore), relief due to excavations and pipe flotation due to soil inundation. In recent years, the subject of offshore pipelines located in arctic environments has been a topic of renewed interest, where the interaction between ice keels and seabed sediments can induce interaction between offshore pipelines and the surrounding soils. Literature pertaining to soil–pipeline interaction studies is quite extensive and no attempt will be made to provide a comprehensive survey. The articles and volumes by Ariman et al. (1979), ASCE (1980), Shibata et al. (1980), Smith (1981), Pickell (1983), Jeyapalan (1985), Selvadurai (1985; 1991), Selvadurai and Lee (1981; 1982), Selvadurai and Shinde (1993), Selvadurai et al. (1983), Bennett (1988) and Jeyapalan and Jeyapalan (1994) can be consulted for more complete literature surveys.

In a comprehensive modelling of frost heave-induced soil–pipeline interaction, adequate consideration should be given to the various time-dependent thermo-hydro-mechanical phenomena associated with the constitutive behaviour of the soil, the structural response of the pipeline and the constitutive behaviour of the soil–pipeline interface that transmits interactions between the buried pipeline and the surrounding soil. Such an all encompassing treatment of a soil–pipeline interaction problem is a difficult task. For this reason, a variety of analytical and computational procedures of varying complexity have been developed for the modelling of soil–pipeline interaction. These range from simplified models of soil behaviour that represent the soil response in terms of one-dimensional spring elements (Nixon et al., 1983; Ladanyi and Lemaire, 1984; Selvadurai, 1985; Selvadurai and Lee, 1982; Selvadurai et al., 1990) to more complex models which can accommodate the three-dimensional continuum response of the soil (Selvadurai, 1988; Selvadurai and Pang, 1988; Selvadurai and Shinde, 1993). The modelling of the pipeline also follows similar lines. In elementary treatments, the pipeline is modelled as a flexible beam which possesses axial, flexural, shear and torsional stiffnesses and, in more advanced treatments, the pipeline is modelled as a cylindrical shell that can exhibit complex nonlinear constitutive phenomena. The interface, similarly, can be assigned smooth, bonded, frictional and other nonlinear constitutive responses (Selvadurai and Boulon, 1995).

The primary objective of this paper is to develop a computational approach to the study of soil–pipeline interaction which is induced by frost heave processes. In recent years, there has been considerable interest in the utilization of pressurized long-distance pipelines for the transportation of natural gas from the Arctic to customers in the south. One such method consists of transportation of the gas in a chilled state at below freezing temperature. The rationale for the chilled gas pipeline is to avoid thawing of ice-rich soil which could result in long-term environmental damage of the permafrost regions. The presence of a chilled gas pipeline in a frost-susceptible soil can result in the gradual development of a zone of frozen soil around the pipeline (Slusarchuk et al., 1978; Myrich et al., 1982; Dallimore and Williams, 1984; Konrad and Morgenstern, 1984; Bahmanyar and Harrison, 1985; Nixon, 1987a,b; Svec, 1989; Shen and Ladanyi, 1991; Selvadurai and Shinde, 1993). If such frost heave development occurs in a relatively uniform fashion, the impact on the buried pipeline is expected to be negligible. With long-distance buried pipelines, such uniformity of frost heave development is rarely possible. Due to the variability in the frost susceptibility and frost heave characteristics, it is likely that a greater variability in the frost heave will be observed along a pipeline. The nonuniformity in the frost heave can impose severe stresses and strains in a buried pipeline. In particular, the effects are accentuated in situations where there is a sudden demarkation between frost susceptibility criteria for adjacent soil regions which can either be frozen or unfrozen, and additional heave induced by a chilled gas pipeline will initiate a net long-term frozen soil–pipeline interaction process. The environmental impact of damage and rupture of oil and gas pipelines can be quite severe. Recent reports from the Russian arctic city of Usinsk indicate that leakage of oil from aging pipelines is a problem of major environmental concern (York, 1994).

This paper presents a continuum approach to the study of interaction between a buried pipeline and the surrounding soil, which can be induced by the discontinuous heave associated with a frost-susceptible zone. The interaction is induced by the time-dependent growth of a frost bulb and the pressure-dependent expansion of the freezing action of a frost-susceptible soil. The interaction is also highlighted by the anchoring action of the pipeline at a prefrozen zone that does not experience any volumetric expansion. The development of frost heave
within a frost-susceptible soil is modelled by a three-dimensional continuum modelling of the coupled processes of heat conduction and moisture migration due to cryogenic effects discussed in a companion paper (Selvadurai et al., 1999). The frost heave modelling is coupled with a continuum model of soil–pipeline interaction where the pipeline is represented as a beam element which exhibits axial, shear and flexural stiffness characteristics. The constitutive behaviour of the frozen soil is modelled by elastic–viscoplastic behaviour and the unfrozen soil is modelled as an elastic material. The computational modelling is applied to the study of the frost heave-induced soil–pipeline interaction at a discontinuous frost-heave zone. In particular, the geometry of the pipeline and soil characteristics are similar to those encountered in the pipeline freezing experiment conducted at the “Station de Gel” at the Centre de Géomorphologie du Centre National de la Recherche Scientifique, Caen, France. The results of the computational modelling are compared with experimental observations derived from these large-scale experiments.

2. Constitutive modelling of frozen and unfrozen soil

The constitutive behaviour of frozen soils is largely characterized by their time-dependent stress–strain response. Extensive experimental, theoretical and computational modelling of the behaviour of frozen soils has been accomplished over the past two decades. A comprehensive account of current developments and references to past work is given in the recent volume by Andersland and Ladanyi (1994). The time-dependent behaviour of frozen soils can be modelled by appeal to creep models or viscoplastic models. The creep behaviour of frozen soils has been investigated quite extensively by Ladanyi (1972), Tsytovich (1975), Andersland and Anderson (1978), Ladanyi (1981), Morgenstern (1981), Phukan (1985), Parameswaran (1987), Phukan (1993) and Ladanyi (1997). In such studies, the focus is primarily with the modelling of primary creep and steady state creep. Power law creep models have largely featured in such developments. In recent years, the developments have been extended to include tertiary creep or creep damage in the frozen soil (Puswewala and Rajapakse, 1993; Selvadurai and Hu, 1995) and brittle fragmentation (Selvadurai and Sepehr, 1997). The second approach to the modelling of creep behaviour of frozen soils involves the use of viscoplastic modelling which employs failure criteria for such soils and a fluidity parameter (Perzyna, 1966; Mroz and Norris, 1982; Zaretskiy, 1993). Both these approaches will have their applicability to the constitutive modelling of frozen soils depending upon the characteristics of the soils in question. The choice of the specific constitutive modelling ultimately has to be decided via experimentation. With reference to the soil–pipeline interaction at a discontinuous frost heave generation zone, it is clear that the frozen soil can be subjected to intense stressing which can initiate failure. The failure processes that can be activated will be consistent with the rate at which frost heave generation takes place. It is conjectured that the constitutive modelling which accounts for primary, secondary and tertiary effects will be more applicable to the slow generation of the discontinuous heave, whereas, constitutive modelling, which accounts for viscoplastic effects will be more suitable for situations where the discontinuous heave is expected to occur relatively rapidly so that the fabric of the frozen soil has insufficient time to allow for relaxation of stresses within it. For the purposes of this study, attention will be restricted to the consideration of elastic–viscoplastic effects. The incremental strain rates in the frozen soil are assumed to be composed of the elastic, viscoplastic and heave components such that

\[
d\hat{e}_{ij} = d\hat{e}^{(e)}_{ij} + d\hat{e}^{(vp)}_{ij} + d\hat{e}^{(b)}_{ij},
\]

In the ensuing, the governing equations for both elastic and viscoplastic phenomena will be summarized.
2.1. Elastic modelling

It is assumed that the soil in the frozen region and the soil in the unfrozen region can exhibit elastic responses that can be characterized by a homogeneous isotropic linear elastic model. The incremental isotropic linear elastic stress rate–strain rate law can be represented by the relationship

\[ d\sigma_{ij} = D_{ijkl} d\epsilon_{kl}^{(i)}, \]

where \( D_{ijkl} \) is the elasticity tensor which can be written in the form

\[ D_{ijkl} = \frac{E_i \nu_j}{(1 + \nu_i)(1 - 2\nu_i)} \delta_{ij} \delta_{kl} + \frac{E_s}{2(1 + \nu_i)} (\delta_{ik} \delta_{jl} + \delta_{jk} \delta_{il}) \]

and \( E_i, \nu \) are, respectively, the elastic modulus and Poisson’s ratio for either the frozen or unfrozen soils and \( \delta_{ij} \) is Kronecker’s delta function.

2.2. Viscoplastic modelling

The extent of failure development within the frozen soil is expected to be influenced by the rate of heave evolution in the discontinuous frost-heaving process. For this reason, the modelling of failure should incorporate both provision for failure and rate effects. The most convenient constitutive model which accommodates both phenomena is a viscoplastic model. Theories of viscoplasticity have been used quite successfully to investigate rate-dependent failure evolution in geomaterials (see, e.g., Perzyna, 1966; Zienkiewicz and Corneau, 1974; Owen and Hinton, 1980; Mroz and Norris, 1982; Zaretskiy, 1993).

The viscoplastic constitutive model basically retains the notion of failure generation in the sense of a classical theory of plasticity, but has the facility to accommodate rate effects by the introduction of a fluidity parameter. As a consequence, much of the methodologies applicable to problems in classical plasticity theory can be adopted in its formulation. The initiation of viscoplastic effects is governed by the attainment of a stress state which satisfies a yield condition. In this study, the onset of viscoplastic effects is governed by a rate-dependent yield condition of the form

\[ F(\sigma_{ij}, \epsilon_{ij}^{(vp)}) - F_0 = 0, \]

where \( F_0 \) is a characteristic yield stress of the material. It is assumed that viscoplastic flow occurs only at the attainment of the yield criterion (Eq. 4). The generalized form of Eq. 4 takes into consideration the influence of strain hardening/strain softening and other history-dependent phenomena. In this particular study, we employ the Mohr–Coulomb yield criterion to represent \( F \); we have

\[ F = \frac{I_1}{3} \sin \varphi + \sqrt{J_2} \left( \cos \theta - \frac{1}{\sqrt{3}} \sin \theta \sin \varphi \right) - c \cos \varphi = 0, \]

where

\[ \theta = \frac{1}{3} \sin^{-1} \left( -\frac{3\sqrt{3}}{2} \frac{J_2}{J_2^1/2} \right); \left( -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6} \right) \]

\[ I_1 = \sigma_{kk}, \quad J_{2} = \frac{1}{2} \sigma_{ij}^D \sigma_{ij}^D, \quad J_{3} = \frac{1}{3} \sigma_{ij}^D \sigma_{ij}^D \]

and \( \sigma_{ij}^D \) is the stress deviator tensor defined by

\[ \sigma_{ij}^D = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}. \]
The viscoplastic strain increment rate can be obtained from a flow rule of the form
\[ d\epsilon_{ij}^{(vp)} = \gamma \langle \Phi(F) \rangle \frac{\delta F}{\delta \sigma_{ij}} d\lambda, \] (8)
where \( \gamma \) is the fluidity parameter, \( d\lambda \) is a scalar parameter and
\[ \langle \Phi(F) \rangle = \Phi(F) \text{ if } F > 0 \] (9)
and
\[ \langle \Phi(F) \rangle = 0 \text{ if } F \leq 0 \] (10)
The function of \( \Phi(F) \) is a positive monotonically increasing function. Several forms for \( \Phi(F) \) have been
proposed in the literature. Examples of these include
\[ \Phi(F) = \left( \frac{F - F_0}{F_0} \right) \] (11)
and
\[ \Phi(F) = \left( \frac{F}{F_0} \right)^n \] (12)
where \( n \) is an arbitrary constant. In this study, the power law representation of \( \Phi(F) \) is chosen.

In order to define the viscoplastic model adopted in this study, the following material parameters need to be
specified; the fluidity parameter \( \gamma \); the Mohr–Coulomb strength parameters \( c, \varphi \); and the exponent \( n \).

3. Modelling of pipeline behaviour

Pipelines are thin-walled cylindrical shells in which the mechanical responses are governed by generalized
deformations which take place both along the pipeline and at its cross-section. The modelling of a pipeline as a
shell will account for longitudinal bending of the pipeline and for ovalization about the cross-section. The
consideration of such effects is certainly possible and should be considered in an all encompassing treatment of
a frost heave-induced soil–pipeline interaction problem. This paper considers a relatively elementary model of
the pipeline behaviour which can be used to test the overall trends predicted by the frost heave-induced
soil–pipeline interaction.

3.1. A beam model of the pipeline

We consider a cylindrical shell of a pipeline of outer diameter \( d \) and wall thickness \( t \). The elastic modulus
and Poisson’s ratio for the pipe material are denoted by \( E \) and \( \nu \). The pipeline is modelled as a Bernoulli–Euler
beam which possesses flexural, axial, shear and torsional stiffnesses consistent with the effective cross-sectional
area of the pipeline. Such a model has been successfully used by Selvadurai and Shinde (1993) to examine the
problem of soil–pipeline interaction at a discontinuous frost heave zone in which the frost heave generation is
approximated via a geothermal simulator Nixon, 1987a,b. In the computational modelling, the beam element is
coupled to the continuum pipeline element at nodal points common to both the one-dimensional pipeline
element and the continuum element. The generalized nodal displacements and the generalized nodal forces can
be written in the forms
\[ [u]^T = [u_1, u_2, u_3, \theta_1, \theta_2, \theta_3] \] (13)
and

$$[F]^T = [F_1, F_2, F_3, M_1, M_2, M_3],$$

(14)

respectively, where $T^*$ denotes the transpose. The matrix equation governing the behaviour of the beam element is given by

$$[u] = [K][F],$$

(15)

where

$$[K] = \begin{bmatrix}
K_{11} & K_{12} \\
K_{12}^* & K_{22}
\end{bmatrix}$$

(16)

where the submatrices are

$$K_{11} = \begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L^3} & 0 & 0 & \frac{6EI}{L^2} & \\
0 & 0 & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \\
0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & 0 & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \\
0 & \frac{6EI}{L^2} & 0 & 0 & \frac{4EI}{L}
\end{bmatrix}$$

(17)

$$K_{22} = \begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L^3} & 0 & 0 & -\frac{6EI}{L^2} & \\
0 & 0 & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \\
0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & 0 & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \\
0 & -\frac{6EI}{L^2} & 0 & 0 & \frac{4EI}{L}
\end{bmatrix}$$

(18)
where $E$ is the Young’s modulus, $G$ is the shear modulus; $I$ and $J$ are the second moment of area about the axis of flexure and the polar moment of inertia, respectively, $l$ is the length of the beam element and $A$ is effective cross-section area. Since a node in the continuum element model has three degrees of freedom and the beam element has six degrees of freedom per node, a double node is required to include three displacements and three rotations for the nodes in the beam. Consequently, the beam element is a four-noded one-dimensional element. The overall accuracy of the representation of the pipeline element by a beam element with connectivity along a line of nodes has been investigated by Selvadurai and Shinde (1993) and will not be further discussed. It is sufficient to note that when the relative flexural stiffness of the soil–pipeline system $R = \frac{EI}{Ed}$, where $d$ is the outer diameter of the pipeline and $I$ is the moment of inertia of the pipeline shell of diameter $d$ and wall thickness $t$ satisfies the constraint

$$R \epsilon (10^3, 10^5),$$

the results derived from the elementary beam model correlates well with the results derived from both the shell model and the solid element model where the pipeline is represented by elastic solid elements with the same outside diameter as the pipeline, but with an “equivalent elastic modulus” which gives an identical flexural stiffness.

4. Computational modelling of soil–pipeline interaction

The computational modelling of frost heave development which accompanies the coupled processes of heat conduction and moisture transport was fully discussed in the companion article (Selvadurai et al., 1999). We now extend the computational modelling to accommodate constitutive responses of the soils in the frozen and unfrozen regimes and the flexural response of the buried pipeline. The mechanism which drives the soil–pipeline interaction process is the frost heave and the rate of evolution of the equivalent nodal forces increment vector $d[R]$ (defined by Eq. (23) of Selvadurai et al. (1999)) induced by the volumetric expansion of frost action process.

4.1. The finite element model

For the purposes of completeness, only the essential details of the finite element procedure are documented. The solid continuum region is modelled by eight-noded three-dimensional solid isoparametric elements. The
displacement vector \( \{u\} \) within the material domain can be represented by the discretized nodal displacement vectors \( \{u_i\} \) as

\[
\{u\} = [N]\{u_i\},
\]

(21)

where \([N]\) is the matrix of element shape functions. The strain vector can be written in the matrix form

\[
\{ \varepsilon \} = [\nabla]\{u\},
\]

(22)

where \([\nabla]\) is a differential operator matrix relating displacements to strains. It follows that

\[
\{ \varepsilon \} = [\nabla][N]\{u_i\} = [B]\{u_i\},
\]

(23)

where \([B]\) is the strain-displacement matrix.

From Eq. (1), the incremental stress rate in Eq. (2) can be re-written as

\[
d\sigma_{ij} = D_{ijkl}d\varepsilon_{kl} - d\mathbf{P}_{ij},
\]

(24)

where the incremental stress rate \(d\mathbf{P}_{ij}\) is due to volumetric expansion during freezing and viscoplastic flow, which is given by

\[
d\mathbf{P}_{ij} = D_{ijkl}[d\varepsilon^{(b)}_{kl} + d\varepsilon^{(vp)}_{kl}],
\]

(25)

where \(d\varepsilon^{(b)}_{kl}\) is the volumetric expansion strain rate due to frost heave. In matrix form, the stress rate–strain rate relationship can be written as

\[
d\{\sigma\} = [D][B]d\{u\}_e - d\{\mathbf{P}\},
\]

(26)

where \(D\) is the conventional linear elastic matrix (see, e.g., Zienkiewicz and Taylor, 1989) and the stress rate, strain rate and the initial stress rate vectors, respectively, take the forms

\[
d\{\sigma\}^T = \begin{pmatrix}
d\sigma_{xx} & d\sigma_{xy} & d\sigma_{xz} \\
d\sigma_{yx} & d\sigma_{yy} & d\sigma_{yz} \\
d\sigma_{zx} & d\sigma_{zy} & d\sigma_{zz}
\end{pmatrix}
\]

(27)

\[
d\{\varepsilon\}^T = \begin{pmatrix}
d\varepsilon_{xx} & d\varepsilon_{xy} & d\varepsilon_{xz} \\
d\varepsilon_{yx} & d\varepsilon_{yy} & d\varepsilon_{yz} \\
d\varepsilon_{zx} & d\varepsilon_{zy} & d\varepsilon_{zz}
\end{pmatrix}
\]

(28)

\[
d\{\mathbf{P}\}^T = \begin{pmatrix}
d\mathbf{P}_{xx} & d\mathbf{P}_{xy} & d\mathbf{P}_{xz} \\
d\mathbf{P}_{yx} & d\mathbf{P}_{yy} & d\mathbf{P}_{yz} \\
d\mathbf{P}_{zx} & d\mathbf{P}_{zy} & d\mathbf{P}_{zz}
\end{pmatrix}
\]

(29)

\[
[B]^T = \begin{bmatrix}
\frac{\partial N}{\partial x} & 0 & 0 & \frac{\partial N}{\partial z} \\
0 & \frac{\partial N}{\partial y} & 0 & \frac{\partial N}{\partial z} \\
0 & 0 & \frac{\partial N}{\partial z} & \frac{\partial N}{\partial y}
\end{bmatrix}.
\]

(30)
\( \{d\vec{u}\}_e \) is the elemental \((e)\) incremental nodal displacement vector and \( N \) is the matrix of shape functions of the element (Zienkiewicz and Taylor, 1989). For an incremental equilibrium relationship, the discretized form is given by

\[
\sum_{e} \delta d(\vec{u})_e^T [K_e] d(\vec{u})_e - d(\vec{R})_e = 0,
\]

where \( K_e \) is the elemental stiffness matrix given by

\[
[K_e] = \int_{\Omega_e} \{B\}^T [D] \{B\} \, d\Omega
\]

and \( d(\vec{R})_e \) is the rate of the incremental load vector due to frost heave strains and viscoplastic strains. The rate of the incremental load \( d(\vec{R})_e \) given in Eq. (28) can be written as

\[
d(\vec{R})_e = \int_{\Omega_e} \{B\}^T [D] \left[ d\left( \begin{pmatrix} \varepsilon^{(b)}_e \\ \varepsilon^{(vp)}_e \end{pmatrix} \right) + d\left( \begin{pmatrix} \dot{\varepsilon}^{(b)}_e \\ \dot{\varepsilon}^{(vp)}_e \end{pmatrix} \right) \right] \, d\Omega
\]

with

\[
\left[ d\left( \begin{pmatrix} \varepsilon^{(b)}_e \\ \varepsilon^{(vp)}_e \end{pmatrix} \right) + d\left( \begin{pmatrix} \dot{\varepsilon}^{(b)}_e \\ \dot{\varepsilon}^{(vp)}_e \end{pmatrix} \right) \right] = d\varepsilon^{(b)} \delta_{ij} + \gamma(\Phi(F)) \frac{\partial F}{\partial \sigma_{ij}} \, d\lambda
\]

with Eq. (31) being applied. From the summation of all element responses, defined by Eq. (28) we obtain

\[
[K] d(\vec{u}) = d(\vec{R}),
\]

which is an incremental equation for the displacement rate where \([K]\) is the stiffness matrix, \(d(\vec{u})\) is the incremental rate of the displacement and \(d(\vec{R})\) is the incremental rate of the local vector due to frost heaving and viscoplastic strains.

### 4.2. Time integration scheme

The time integration is performed by using a Euler integration scheme such that for the time station \(t_n+1\)

\[
\left[ d\left( \begin{pmatrix} \varepsilon^{(b)}_e \\ \varepsilon^{(vp)}_e \end{pmatrix} \right) + d\left( \begin{pmatrix} \dot{\varepsilon}^{(b)}_e \\ \dot{\varepsilon}^{(vp)}_e \end{pmatrix} \right) \right]_{n+1} = \left[ d\left( \begin{pmatrix} \varepsilon^{(b)}_e \\ \varepsilon^{(vp)}_e \end{pmatrix} \right) + d\left( \begin{pmatrix} \dot{\varepsilon}^{(b)}_e \\ \dot{\varepsilon}^{(vp)}_e \end{pmatrix} \right) \right]_n (\Delta t),
\]

where \(\Delta t\) is the time increment. In conventional time integration involving dynamic effects, the \(\theta-\) Wilson technique provides an unconditionally stable integration scheme. In viscoplastic analysis, however, the accuracy of the integration procedure can deteriorate with an increase of the time step. Therefore, a limitation on the time step should be imposed in the time stepping procedure. The theoretical restriction in \(\Delta t\) can be written in the following form (see, e.g., Corneau, 1975):

\[
\Delta t \leq \frac{4(1 + \nu)(1 - 2\nu)e \cos \varphi}{\gamma (1 - 2\nu + \sin^2 \varphi) E_s}.
\]

In addition to the time-integration, the numerical procedure also involves an iteration procedure to account for nonlinear effects. Since the function \(d(\vec{R})\) is dependent on the current state of stress in the soil medium, an iterative technique must be employed to obtain the correct value of \(d(\vec{R})\). Therefore, the iterative version of Eq. (35) can be rewritten in the form

\[
[K] \{d\vec{u}\}^{(i)} = \{d\vec{R}\}^{(i-1)},
\]

where the superscript \((i)\) indicates the number of iterations and \(d(\vec{R})^{(0)}\) is the result at the end of the previous time step. The iteration is performed until the change in displacements in two successive iterations is less than a prescribed acceptable limit. This accuracy is usually set at \(10^{-3}\)%. 

5. Experiments at the Caen test facility

In this section, we present a brief description of the large-scale test facility at Station de Gel at Le Centre de Geomorphologie at Caen, France where the Canada–France pipeline experiment was conducted. More complete descriptions of these experiments are given by Dallimore and Williams (1984) and Dallimore (1985).

The Caen test facility consists of a temperature-controlled hall which is 18 m long, 8 m wide and 5 m high. The test facility contains a trough which can be filled to a depth of 2 m. The base of the trough can isolate the
thermal and hydraulic regime within it from the natural ground conditions. The test trough is filled with soil to a depth of 1.75 m. The objective of the test was to create a sharp contrast in the frost susceptibility of soils used in the experiment. To achieve this, two separate soils were used. The Caen silt was used to model the highly frost-susceptible soil (A grain size analysis indicated that Caen silt consists of 13%–20% clay, 65%–75% silt and 10%–20% sand.). To model the soil with low frost susceptibility, the SNEC sand was used (A grain size analysis indicated that the SNEC sand consisted of less than 10% silt and 80%–90% sand.). The Caen silt and the SNEC sand each occupied one-half of the test trough. The test pipeline consisted of a steel pipeline of length 18 m, diameter 273 mm and wall thickness 0.5 mm. The pipeline had an independent refrigeration system. The instrumented test pipeline was buried in a trench with a depth of 33 cm excavated along the entire 18-m length of the pipeline and backfilled with identical soils. In order to model a relatively long-distance pipeline, both ends of the pipe were kept free of any constraints. The longitudinal and transverse cross-sections of the test facility are shown in Fig. 1.

The freezing experiment consisted of two major stages. The first stage commenced on September 1982 with four freezing periods. The objective of this test was to examine the behaviour of a chilled pipeline located at the intersection between two initially unfrozen soils of widely different frost susceptibilities. The second experiment was designed to investigate the behaviour of a chilled pipeline which was located at the transition zone between prefrozen and unfrozen frost-susceptible soil.

In the first experiment, the initially unfrozen soils were subjected to four periods of freezing. The first period continued from 21st September 1982 to 8th June 1983, lasting approximately 8.5 months. The air temperature in the hall was maintained at $-0.75^\circ C$ and the temperature in the pipe was maintained at $-2^\circ C$. A 4-month thawing was achieved by raising the ambient temperature in the hall to $+4^\circ C$. During this thawing sequence on 17th October 1983, it was observed that nearly all of the annulus of frozen soil formed during the first freezing period had melted. The second freezing period commenced on 17th October 1983. All conditions associated with the test were kept the same as in the first freezing sequence except that the pipe temperature was maintained at $-5^\circ C$. The duration of the test lasted 450 days. The freezing commenced by the lowering of the air temperature in the hall to $-0.75^\circ C$ and by the circulation of gas in the buried pipeline at $-5^\circ C$. The moving freezing front was formed by a combination of freezing front which progressed downward as a result of the cooling in the hall region. A second radially moving freezing front was initiated as a result of cooling at the pipe. A second stage of a freezing test was carried out in the test facility between 1990 and 1993. In this test, a pipeline was embedded at a transition region between a prefrozen soil and an unfrozen soil. The objective of the test was to examine the forces that would be generated on a pipeline which was anchored by a prefrozen soil. In order to allow comparison between the previous experiments, the composition of silt, the size of pipe, the depth of burial and water influx were kept the same. Prior to the start of the experiment (as indicated by the commencement of pipeline cooling), the prefreezing of the SNEC sand lasted about 230 days. After this period, the ambient air temperature in the facility was maintained at $-0.75^\circ C$ and the cooling temperature in the pipeline was maintained at $-5^\circ C$. This temperature specification was maintained for 215 days. From day 215 to day 256, the temperature in the pipe was lowered to about $-8.5^\circ C$. This was followed by a relaxation period and a second freezing period. Since the results derived from the experiments covering both the initial freezing of the unfrozen silt and sand, the freezing of prefrozen sand and the unfrozen silt are well documented, computational modelling will cover both scenarios. The results derived from the experimental investigations are fully documented by Carlson (1994) and will be presented, where appropriate, in the subsequent sections.

6. Computational modelling of the buried pipeline problem

The computational modelling procedure outlined in Section 5 will now be applied to examine the frost heave-induced soil–pipeline interaction at a discontinuous frost heave zone. The dimensions of the prismatic soil region modelled, measured $18 \text{ m} \times 18 \text{ m} \times 1.75 \text{ m}$. The finite element discretization shown in Fig. 2 consists of 360 continuum elements and 10 beam elements. (It is noted that symmetry of the problem is not
Fig. 2. The finite element discretization of the soil–pipeline system in the Caen experiment.

invoked in the discretization procedure with a view to examining, in future studies, the influence of backfill inhomogeneities.) A variety of mechanical and physical properties are used for the computational modelling of the problem. These will be discussed very briefly in the ensuing sections.

6.1. The buried pipeline

The pipeline used in the experiment is made of steel with Young’s modulus 200 GPa and Poisson’s ratio 0.30. The external diameter of the pipeline was 273 mm and the wall thickness was 5 mm.

6.2. The Caen silt

The self weight of the Caen silt is taken as 15 kN/m³ and the initial moisture content is taken as 40%. The thermal conductivity of the silt is assumed to be a constant;

\[
\lambda = 0.65 \text{ Wm}^{-1} \text{°C}^{-1}.
\]

The hydraulic conductivity of the frozen Caen silt is estimated from the relationship

\[
k = \begin{cases} 
1.075C \times 10^{-9} e^{23.99T}; & -0.3°C < T < T_f \text{ m/s} \\
8.0499C \times 10^{-13}; & T \leq -0.3°C \text{ m/s} 
\end{cases}
\]

where \( C = 2.75 \).

The Young’s modulus of the unfrozen silt is taken as 11.2 MPa and the Poisson’s ratio is assumed to be 0.40. The Young’s modulus of frozen silt is considered to be a function of the temperature below the freezing point. Shen and Ladanyi (1991) suggest the following empirical variation:

\[
E_s = 400 \left( \frac{T}{T_f} \right)^{0.636} \text{ MPa},
\]

where \( T_f = -1°C \) is the reference temperature.
In order to model viscoplastic failure of the frozen silt, it is necessary to specify the strength parameters associated with the Mohr–Coulomb failure criterion and the fluidity parameter $\gamma$. To date, there are no published experimental results either for the Caen silt or for the SNEC sand which will enable the direct determination of these parameters. The approach adopted in the computational modelling exercise is to assign plausible values for these parameters from results available in the literature.

The peak values of the shear strength parameters are as follows:

$$c = 1500 \text{kN/m}^2; \quad \varphi = 30^\circ.$$  \hspace{1cm} (42)

It is assumed that the frozen soil experiences softening beyond the attainment of the peak strength. The residual shear strength parameters are as follows:

$$c_r = 500 \text{kN/m}^2; \quad \varphi_r = 15^\circ.$$  \hspace{1cm} (43)

The fluidity parameter $\gamma$ is assigned the value

$$\gamma = 5 \times 10^{-7} \text{s}^{-1}.$$  \hspace{1cm} (44)

This value of the fluidity parameter can give approximately the equivalent strain rate as with the primary creep equation $\dot{e} = AC\sigma^{\frac{1}{2}} \dot{t}$, where the creep parameters for a typical frozen soil are $A = 1 \times 10^{-17}$; $B = 4$; $C = 0.1$ (Klein, 1979). The value of the fluidity parameter is assumed to be the same at both peak and residual stress levels.

6.3. The SNEC sand

The self weight of the SNEC sand is taken as 18.5 kN/m³ and the initial moisture content is taken as 22.7%. The thermal conductivity of the sand is assumed to be a constant;

$$\lambda = 2.2 \text{W/m} \cdot \text{°C}^{-1}.$$  \hspace{1cm} (45)

The permeability of the frozen SNEC sand is estimated from a relationship of the type defined by Eq. (40) except that the constant $C$ in Eq. (40) is set equal to 0.001.

The Young’s modulus of the unfrozen sand is taken as 20 MPa and Poisson’s ratio is assumed to be 0.30. The Young’s modulus of the frozen sand is assumed to be identical to the value derived from the result (Eq. (41)). The viscoplasticity properties of the frozen sand are assumed to be identical to the values applicable to Caen silt prescribed in Section 5.

7. Computational results

We shall present, separately, computational results derived for the soil–pipeline interaction problems involving (i) the soil region consisting of unfrozen silt and sand and (ii) the soil region consisting of unfrozen silt and prefrozen sand.

7.1. The pipeline experiment involving unfrozen silt and sand

The computational modelling of the soil–pipeline interaction problem examined in this study focuses on the simulation of the frost heave-induced interaction associated with the refreezing problem where the air temperature of the test facility is maintained at $-0.75^\circ$C and the temperature of the pipeline is maintained at $-5^\circ$C. Attention will be focused on the presentation of numerical results which can be compared with equivalent results derived from the large-scale experiment.
7.1.1. Temperature distributions

Fig. 3 illustrates a comparison of temperature distributions within the frozen Caen silt for lapsed times of 21 days, 105 days and 379 days derived from the computational scheme with equivalent results derived from the

![Diagram of temperature distributions](image)

Fig. 3. Temperature distributions within the frozen silt. Comparison between computational estimates and experimental results.
experiment. Similar comparative results for the experimental section located in the SNEC sand are shown in Fig. 4. In the experimental data, only the zero degree isotherm is recorded. The trends indicated by the computational model are consistent with the experimental data.

Fig. 4. Temperature distributions within the frozen sand. Comparison between computational estimates and experimental results.
Fig. 5. Frost penetration beneath the pipeline in the Caen experiment. Comparison between computational estimates and experimental results.

Fig. 6. Computational estimates for the surface heave contours due to frost heave generation by ambient cooling and pipeline cooling.
7.1.2. Frost heave development

During the experiment, the depth of frost penetration beneath the pipeline was also measured at the sections A–A and B–B located within the Caen silt and SNEC sand regions. Similar estimates were also derived from
the present computational model. Fig. 5 illustrates the comparison between the experimental values for the frost penetration in the Caen silt and SNEC sand regions. It is evident that there is an accelerated growth of the frost penetration beneath the pipeline at both the silt and sand regions. After approximately the initial 110 days, the development of frost heave stabilizes to a very low rate. Both these experimental observations are borne out in the results derived from the computational model.

The contours of the frost heave over the entire surface of the test facility were determined in the experiment. The results from the computational modelling were therefore used to determine the surface frost heave contours. Fig. 6 illustrates the contours of frost heave measured at the soil surface. These results are also presented in isometric view in Fig. 7. The computations were performed to simulate the time duration of 227 days. The results for the contours of experimentally derived frost heave in both the surface of the Caen silt and SNEC sand are shown in Fig. 8. The results of the computational model indicate trends which are consistent with the numerical computations. Comparisons between experimentally derived results for surface heave and equivalent computational results obtained at specific locations (sections A–A and B–B) are shown in Fig. 9. Here again, good agreement is obtained between the experimental results and computational estimates.

7.1.3. Pipeline response

In the experimental investigations, extensive records were made of the pipeline response. Both the deflections induced in the pipeline during the time-dependent frost heave process and the development of bending stresses along the pipeline were measured. The computational model was used to identify similar results. Fig. 10 illustrates the time-dependent evolution of displacements along the pipeline, at various time intervals, derived from the computational modelling. It is evident that the pipeline displacements are very similar in character to the measured trends in the frost heave evolution. Equivalent results derived from the experimental investigation are shown in Fig. 11. The time-dependent variation of bending stresses along the length of the pipeline determined via the computational modelling is shown in Fig. 12. These variations exhibit

Fig. 9. Time-dependent evolution of surface heave at locations in the Caen silt and SNEC sand. Comparison between computational estimates and experimental results.
Fig. 10. Computational estimates for the time-dependent displacement of the pipeline due to frost heave.

Fig. 11. Experimental results for the time-dependent displacement of the pipeline due to frost heave.
relatively abrupt changes at certain locations of the pipeline particularly in the vicinity of the transition zone. This is primarily due to the relatively coarse mesh refinement adopted in the discretization of the beam elements and the continuum elements in the vicinity of the transition region. Equivalent results obtained from the experiment are shown in Fig. 13. The general trends indicated in the computational modelling are consistent with the experimental observations. It must be emphasized that the numerical results are particularly sensitive to the mesh refinement at the transition zone (Selvadurai and Shinde, 1993).

7.2. The pipeline experiment involving prefrozen sand

Attention is now focused on the computational modelling of the experiment where the SNEC sand is in a prefrozen state and the subsequent lowering of the ambient air temperature in the hall region to $-0.75^\circ C$ and the cooling of the pipeline to $-5^\circ C$. The duration of the experiment is varied between 246 days and 257 days to suit observations.

7.2.1. Temperature distributions

The computational and experimentally derived temperature distributions for the situation involving the prefrozen sand closely follow the temperature distributions described previously (Figs. 3 and 4) for the unfrozen soil regimes. The temperature profiles within the Caen silt and the prefrozen SNEC sand are largely governed by the air temperature within the test facility and the temperature of the pipeline. The changes in the heat
conduction parameters resulting from the prefreezing of the SNEC sand appears to have little influence on the experimentally derived temperature fields. The computational scheme assumes that the thermal conductivities and heat capacities of the Caen silt and SNEC sand are uninfluenced by any prefreezing (see, e.g., Eqs. (39) and (42)).

7.2.2. Frost heave development

Fig. 14 illustrates the variation in the frost heave-induced displacement at various locations of the pipeline, derived from the computational model. Analogous results obtained from the experiment are shown in Fig. 15. Again, in general, the trends indicated by the computational model are consistent with the results derived from the experiments. The end of the pipeline which is located at the unfrozen Caen silt region will experience continuous heave. The computational modelling indicates that the section of the pipeline which is located at the interface also experiences heave, but at a different rate to that observed in the experiments. Similar comments apply to the section of the pipe which is located at the prefrozen end. Figs. 16 and 17 illustrate the results for the surface heave of the soil as derived from the computational model. Analogous results derived from the experiment are shown in Fig. 18. The trends are again reasonably consistent.
Fig. 14. Computational estimates for the variation of frost heave-induced pipe displacement at various locations of the pipeline. (Initially prefrozen SNEC sand.)

7.2.3. Pipeline response

The results for the time-dependent variation in the displacements along the pipeline, derived from the computational modelling are shown in Fig. 19. The results obtained from the experiment are documented in the report by Carlson (1994). They are included (Fig. 20) for the sake of completeness. Fig. 21 illustrates the computational results for the time-dependent distribution of bending moments along the pipeline. Analogous experimental results of the bending moment distribution along the pipeline are presented in the report by Carlson (1994); again the trends indicated in the experiments are consistent with the computational results.

As the frost heave occurs, the pipeline experiences differential movement. This differential movement causes uplift in the prefrozen anchored zone. When the frozen soil has finite strength, the uplift movements can cause yielding of the soil. In regions of the frozen soil subjected to such uplift loads, progressive time-dependent
failure can occur in regions surrounding the pipeline. This is particularly the case if adfreezing is present and the frozen soil has appreciable tensile strength. Fig. 22 illustrates the time-dependent evolution of plastic failure within the prefrozen sand due to frost heave-induced pipe uplift. The general pattern of plastic zone development is consistent with the occurrence of a wedge-type failure mechanism (Selvadurai and Sepehr, 1997).

8. Concluding remarks

The evaluation of the mechanics of frost heave-induced interaction between pipelines and frozen soils is important in instances where buried pipelines are subjected to nonuniform frost heave. The nonuniform frost heave can occur as a result of either spatial variations in the frost susceptibility characteristics of unfrozen soils
or at a transition zone between a prefrozen soil and a frost-susceptible soil. The frost heave can be induced by the freezing action of a pipeline which transports, for example, pressurized chilled gas. A necessary aspect of the frost heave-induced soil–pipeline interaction is the simultaneous consideration of a number of processes including heat conduction within the soil mass, moisture transport within the soil mass, progressive development
Fig. 20. Experimental results for the time-dependent displacement of the pipeline due to frost heave. (Initially prefrozen SNEC sand.)

Fig. 21. Computational estimates for the time-dependent distribution of bending stresses in the pipeline due to frost heave. (Initially prefrozen SNEC sand.)
Fig. 22. Evolution of plastic zones within the prefrozen SNEC sand due to frost heave-induced pipe uplift.

of frost heave, mechanical behaviour of frozen and unfrozen soils and the interaction between the pipeline and frozen soils. The complete coupled behaviour of these processes and the three-dimensional nature of the frozen soil–pipeline interaction at a transition zone makes the analysis a problem of considerable complexity. It is generally accepted that such a complete analysis which takes into account the time-dependent effects of full coupling between heat and mass transfer, frost heave generation and mechanical behaviour of frozen and unfrozen soils is unwarranted (Selvadurai and Shinde, 1993). This is primarily due to the inherent variability in the properties of soils encountered along long-distance pipelines and the difficulties associated with determining the complete hydro-thermo-mechanical parameters associated with fully coupled theories. The alternative approach is to consider weak coupling effects where the heat transfer and moisture transport within the soils are assumed to be dominant processes which are not strongly influenced by the constitutive behaviour of the frozen soils. The ice pressure at the coldest side of freezing fringe is, however, assumed to be equal to local mean stress.

This paper focuses on the development of a computational methodology where (i) the generation of frost heave in frozen and unfrozen soils is determined by a consideration of the heat transfer and moisture migration process, (ii) the modelling of the soil pipeline interaction takes into consideration the elastic and viscoplastic behaviour of the frozen soils, (iii) the generation of frost heave within the frozen regions and (iv) the three-dimensional effects of soil–pipeline interaction at discontinuous frost heave zones.

The process of frost heave generation is modelled by taking into consideration the coupled model of heat conduction and moisture flow in a freezing soil proposed by Shen and Ladanyi (1987). The frost heave generation process is thus governed by the thermal parameters of the frozen soils and hydraulic conductivity of
the frozen soil. This model has been calibrated against experimental data derived from one-dimensional experiments conducted on frost-susceptible soils. The mechanical behaviour of the frozen soils and the unfrozen soils is modelled by appeal to theories of elasticity and viscoplasticity. The choice of these models is only intended to demonstrate the methodology associated with the continuum modelling of the soil-response. The pipeline is modelled as a flexible beam which has flexural, shear and extensional stiffnesses. The finite element procedure developed in the study has been applied to examine the frost heave-induced interaction between the buried pipeline and the frost-susceptible Caen silt and SNEC sand encountered in the large-scale test facility in Caen, France. The objective of the comparison is not to validate the computational methodology advocated in the study. Such an exercise requires the careful evaluation of all input parameters associated with heat transfer, moisture transport modelling, elastic behaviour and viscoplastic effects. The computational methodology is used in conjunction with plausible values of input parameters to obtain solutions to the freezing sequences conducted in the Caen test facility. The results obtained from the computational modelling cover a wide range of responses including the time-dependent temperature variations within the silt and sand regions; the time-dependent evolution of surface (frost) heave, the deflection of the embedded pipeline and the time-dependent variation of flexural moments along the pipeline. The results derived from the computational modelling indicate trends which are remarkably consistent with the experimental observations. These conclusions apply to two types of experiments: the first involves the frost heave generation as a result of cooling of the surface of the soil and cooling associated with the pipeline and the second involves a similar experiment except that the sand region is prefrozen. The performance of the computational modelling is sufficiently encouraging to initiate (i) further calibration exercises and sensitivity studies, (ii) extension of the modelling to include thaw settlement, (iii) adaptation of the basic methodology to the study of pipelines modelled as thin shells and (iv) the consideration of probabilistic computational modelling of frost heave development which is of interest to long-distance buried pipelines located in frost-susceptible soils which can exhibit variable frost heave effects.

References

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