Correlation of joint roughness coefficient and permeability of a fracture

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ABSTRACT

This paper uses a new modeling approach to study fluid transport in an open fracture, which relies on data derived from scanned surfaces of fractures and considers surface variations and velocity Laplacians in two orthogonal directions and the bulk flow rotation. The model predictions are verified against the results for incompressible flow modeling obtained using the procedures available in COMSOL® multiphysics. The model examines the relationship between the joint roughness coefficient (JRC) and permeability of a fracture. A new method is presented to reproduce the surfaces of the fracture using both JRC and the fractal dimension. The JRC-permeability correlation is analyzed in exactly mated surfaces, surfaces that have undergone translations in their plane and independent surfaces to highlight the importance of the relative displacement of fracture surfaces in fluid transport analysis. A shifting threshold from which two translated surfaces can be considered independent is determined. It is shown that even a 62.5-micron translation for an aperture size of 0.1 mm has a noticeable effect on the permeability, which first decreases and then increases because of surface translation; the cubic law consistently underestimates the permeability for small apertures (< 0.1 mm) and moderate to high JRCs (> 10) for independent surfaces. The effect of sample size and contact area ratios are also discussed. The presented model is a first step towards developing a computationally efficient, reasonably accurate model for statistical analysis of the fluid transport properties of a network of stressed fractures.

1. Introduction

The fluid transport in fractures has important applications in geosciences dealing with geothermal energy extraction, hazardous waste disposal, sequestration of greenhouse gases, and enhanced oil recovery. The ultimate goal in these areas of application is to estimate the hydraulic conductivity of a permeable domain having a network of fractures, the basic problem of 3-dimensional fluid transport between two rough surfaces (Fig. 1.a) is an important first step. The literature related to modeling fluid flow in a fracture can be arranged into five categories:

1. The fracture is modeled as a separate domain from the intact rock (Fig. 1.b) with a distinct permeability e.g.,1,2 The fracture permeability is normally estimated using the cubic law e.g.,3 While these models cannot clearly capture the effects of fracture roughness, it is a valuable approach for estimating the bulk permeability of the rock.
2. The fracture permeability is estimated by investigating the 2-dimensional flow between two rough boundaries (i.e., 2D projections of the fracture surfaces in the y-z plane, see Fig. 1.c). This 2D flow is then studied using conventional flow modeling schemes (e.g., finite volume method4) or the assumption of having a successive set of parallel plate passages.5 This simplification neglects the transverse dimensionality of flow in a fracture (i.e., flow in the x-direction in Fig. 1.a) which is normally considered as tortuosity effects in the literature. The second approach (i.e., parallel plate passages) is especially prone to underestimation of the pressure drop because the apparent height (h_a in Fig. 1.c) is considered as the local aperture instead of the real height (h, in Fig. 1.e)6-7.
3. The fluid transport in a fracture has also been studied in the x-y plane in Fig. 1 (Fig. 1.d). In this approach, Navier-Stokes equations are integrated along the aperture's height and equivalent 2D representations are determined e.g.,8,9 and their cited references. Zimmerman and Bodvarsson10 investigated the underlying assumptions and accuracy level of this approach and showed that this approach is specifically restricted to the cases in which the aperture is substantially smaller than the characteristic spatial wavelength of the aperture variations, which is not always satisfied in practice. Even in the case of small apertures compared to spatial characteristic length, since the axis of integration is not always parallel to z-axis, the fracture plane will in fact be a 3D distorted surface and cannot necessarily be mapped in a 2D flat plane, which is not

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considered in the general form of the model. This approach also
considers the apparent height as the local aperture instead of the
real height (Fig. 1.c). Moreover, this approach cannot model the
contact areas and the non-slip boundary condition is not normally
satisfied for the contact areas.

4. To rectify the problem of contact areas, the fracture can be divided
in separate cells that are connected in a spatial network (Fig. 1.e). In
each cell, the average aperture is calculated and the pressure drop is
determined using the cubic law. The cells containing contact areas
(black cells in Fig. 1.e) are removed from the network and the
overall conductivity of the network is calculated (e.g.,10 and the
references cited in3,11). The computational resources required for
this approach are considerably lower than the numerical mesh-
based approaches. However, the accuracy of the results can be
considerably affected by underlying assumptions of the approach,
including a constant aperture in each cell, equal aperture in the x
and y-directions in each cell (Fig. 1.e), neglecting the pressure drop
due to the bulk rotation of the flow, neglecting the Laplacian of
the velocity in the transverse direction, and high spatial frequency of the aperture variations are all
considered in this model. This approach is in fact a 3-dimensional ex-
tension of a 2D modeling approach presented recently for fluid trans-
port in a generalized pore structure.13 It was shown13 that this approach
could estimate the transport properties of a porous structure two orders
of magnitude faster than conventional mesh-based methods with
comparable accuracy. The same approach was extended to study the 3D
fluid transport in a wormhole with an acceptable accuracy.14 Here, the
fracture is divided into separate areas (cells) in a spatial network (si-
milar to Fig. 1.e, but different areas for the fluid transport in the x and
y-directions are considered, see Fig. 2). In each cell, the surfaces ele-
vations are averaged in the transverse direction and a 2D geometry is
obtained (similar to Fig. 1.c). In each cell, the average aperture
is used to calculate the pressure drop due to the bulk rotation of the flow, neglecting the Laplacian of
velocity in the transverse direction (x-direction), and using the ap-
parent height instead of the real height.

5. The fluid transport in a fracture can also be modeled by solving full
3D Navier-Stokes equations using commercial software packages,
such as FLUENT™ e.g.,6,12 While complexity of the flow due to
roughness of the fracture faces requires considerable computational
resources, the results are not always satisfying because the high
spatial frequencies of the surfaces roughness cannot be modeled.12

A new modeling approach for fluid transport between rough
surfaces is presented in this paper, which addresses all of the issues
discussed above; specifically, the effects of transverse flow, real height,
3D distortion of the fracture surface, non-slip boundary conditions of
the contact areas, aperture variations in different directions in each cell,
bulk rotation of the flow, Laplacian of the velocity in the transverse
direction, and high spatial frequency of the aperture variations are all
considered in this model. This approach is in fact a 3-dimensional ex-
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y-directions are considered, see Fig. 2). In each cell, the surfaces ele-
vations are averaged in the transverse direction and a 2D geometry is
obtained (similar to Fig. 1.c). The ratio of pressure drop to flow rate for
this 2-dimensional flow is estimated13 and corrected using the real
surface area determined by integrating the 3D fracture surfaces. The
flow rates in the overall network are determined. The overall pressure
drop of the fracture is finally estimated by adding the pressure drop
increments obtained from transverse velocity effects in the analysis. It
should be mentioned that this model is not always more accurate or
more computationally-efficient than the models reviewed before, as it is
more computationally-efficient to use models of category 4 (network of
cells with constant aperture) and certainly more accurate to solve 3D
Navier-Stokes equations when spatial frequencies of the surfaces

![Fig. 1. Various approaches for modeling fluid transport in a fracture: (a) the real fracture geometry; (b) assuming two separate domains for intact materials and the fracture; (c) 2D projection of the fracture in the y-z plane, real height (h_r) vs. apparent height (h_a); (d) Integration of the governing equations perpendicular to the fracture plane; the aperture variations are characterized using different colors; (e) modeling fracture as a spatial network; aperture differences in separate cells are portrayed using different colors; black cells contain contact areas.](image-url)
roughness are limited. Nevertheless, the model presented here can be considered as a reasonable balance between computationally efficiency and accuracy for general cases, since it enables the statistical analysis of the permeability range of fractures.

The details of the model are presented in the following section. This model is in fact the first step in our efforts towards developing a computationally efficient, reasonably accurate model for statistical analysis of the fluid transport in a network of fractures under stress. The model predictions for various geometries are compared against the modeling results obtained using the COMSOL™ multiphysics computational fluid dynamics package in Section 3. A new approach for numerical representation of the fracture surfaces is presented in the next section. The relationship between permeability and the joint roughness coefficient (JRC) of a fracture for exactly matched surfaces, translated surfaces and independent surfaces are then investigated. These cases are analyzed as sample applications of the proposed model to highlight the aspects of the fluid transport in a fracture that were not discussed previously in the literature. Finally, the effects of sample size and contact area ratio in different cases are studied.

2. Model development

While the real height of an aperture can be defined in a 2-dimensional projection of a fracture (Fig. 1.c), it is a non-trivial task in the case of a 3-dimensional geometry. The total flow has to traverse any cross section of the fracture (i.e., any line that connects two faces) in 2D and as a result, it is reasonable to assume that the pressure drop is proportional to the real height. However, the velocity gradients in the x, y, and z directions in a 3D case result in a more complex flow in which the bulk flow does not necessarily cross local real heights (even if it can be defined). Also, the accuracy, the procedure, and the flow dynamics behind estimating the pressure drop of a 2D passage through tracing its boundaries was discussed previously; however, such analysis is not available for 3D passages for general cases. As a result, tracing the 3D faces of the fracture does not necessarily lead to more accurate results than tracing the 2D locally-averaged lines (similar to Fig. 1.c). However, the Laplacian of the transverse component of the velocity (x-direction in Fig. 1) has to be considered as well. This point will be discussed later in this section.

Here, a structured mesh of locations is defined in the fracture (red points in Fig. 2.a) and the fluid transport between these locations is investigated. For each two adjacent locations in the x and y-directions, a cell is defined that covers the connecting area of the locations (see Fig. 2.b and .c for cells in the y and x-directions, respectively). The fracture face elevation in each cell is averaged in the direction perpendicular to the assumed flow direction and the equivalent 2D projection of the fracture faces in that specific direction is obtained. As an example, for the cells defined in the y-direction (Fig. 2.b), the elevations of the fracture faces are averaged in the x-direction and a 2D representation of the fracture (similar to Fig. 1.c) is obtained. The fluid transport in this 2D representation is then analyzed using the theory presented in and the ratio of pressure drop to flow rate is determined. The utilization of the above-mentioned theory for this specific application consists of (i) estimating the real height for each point on the upper and lower boundary lines, (ii) applying the cubic law for each point based on the real height, (iii) integrating separately for two boundary lines (faces) and (iv) assuming the average value as the overall ratio of pressure drop to flow rate for the cell. It has been shown that this approximation results in errors of less than 10% even in considerably more general cases. While this analysis can also be accomplished theoretically, here the average boundary lines are determined and magnified into a larger hexagonal mesh similar to that presented by Rezaei Niya and Selvadurai and the ratio of pressure drop to flow rate is estimated accordingly.

Once the ratio of pressure drop to flow rate for each cell has been determined, the flow rate distribution in the network is calculated using the Hardy Cross method and the overall pressure drop is calculated. The problem of contact areas is also solved in this approach since the

Fig. 2. The current model: (a) locations defined for fracture discretization; (b) cells defined for flow in the y-direction; (c) cells defined for flow in the x-direction.
clogged passages (cells) are specified and removed from the network.

To improve the accuracy of this approach, several corrections have to be applied to address the following issues: fracture face variations in the transverse direction, variations of Laplacian of transverse velocity and bulk flow rotation. As mentioned before, the elevation of the fracture surfaces is averaged in the transverse direction in each cell to convert the 3D face to a 2D line (similar to Fig. 1.c). As a result, the face variations in the transverse direction are ignored in this case. To solve this issue, the ratio of the real surface area (multiplication of length of the 2D boundary in the width of the cell) has to be included in the analysis. Based on the cubic law, the pressure drop ($\Delta p$) in the ideal case can be determined as

$$\Delta p = -\frac{12\mu Q}{H W} I$$

(1)

where $\mu$, $Q$, $h$, $w$, and $l$ represent the dynamic viscosity of the fluid, the volumetric flow rate, height, width and length of the channel, respectively. Here, the flow rate per unit width in the original equation is replaced by the volumetric flow rate ($Q$) divided by the width of the channel ($w$). There are two ways to model the expansion of faces of a fracture using this equation: through either a length ($l$) increment or a width ($w$) adaptation. The length increment clearly results in a linear increase of the pressure drop. The width adaptation, however, includes an increment of the width of the channel while keeping the ratio of the flow rate to the width constant. In other words, in order to study the effect of the width increment while canceling the effect of a flow rate variation in this process, the $Q/w$ ratio has to be considered. The above equation shows that this parameter also has a linear relationship with the pressure drop. Therefore, it is reasonable to assume that in order to correct the effects of face variations in the transverse direction, the pressure drop needs to be multiplied by the ratio of the real surface area to the projected one.

In the absence of inertial effects, variations of Laplacian of the $y$-direction component of velocity ($\nu$) in the transverse direction ($x$) can result in a non-negligible part of the pressure drop of a fracture. Considering Stokes’ flow equation in the $y$-direction for the cells shown in Fig. 2.b,\textsuperscript{15} we have

$$\frac{\partial \nu}{\partial y} = \mu \left( \frac{\partial^2 \nu}{\partial x^2} + \frac{\partial^2 \nu}{\partial z^2} \right)$$

(2)

Using a 2D projection of the geometry in the $y-z$ plane (Fig. 1.c) and estimating the pressure drop, the effects of the last two terms on the right-hand side of the Eq. (2) are considered. The first term, however, is not considered in this analysis. In other words, the pressure drop caused by viscous losses due to velocity differences between neighboring cells in the $x$-direction is ignored. To estimate this term, the flow rate distribution in the network is determined and the flow rate in each cell is obtained. Then, the overall volume of the fracture in the cell is specified and the average cross section of the fracture in the cell is determined. The average velocity in the cell can then be calculated. Assuming the average velocity of $u_w$ for the cell and average velocities of $\nu_x$ and $\nu_z$ for the neighboring cells in the $x$-direction, the second derivative in the $x$-direction can be determined. The second derivative can be estimated using a finite difference approximation as

$$\frac{\partial^2 \nu}{\partial x^2} = \frac{\nu_x - 2u_w + \nu_z}{(\Delta x)^2}$$

(3)

where $\Delta x$ is the distance between the locations in the $x$-direction. The pressure drop resulting from this term can be obtained as (Eq. (2))

$$\Delta p = -\mu (\Delta x)^2 \frac{\partial^2 \nu}{\partial x^2}$$

(4)

where $\Delta x$ is the distance between the locations in the $y$-direction. This excess pressure drop for each $y$-direction cell (Fig. 2.b) is multiplied by the share of the flow rate in the cell (the cell's flow rate, calculated by neglecting this effect, divided by the overall flow rate) and then added to the overall pressure drop calculated before.

Finally, to improve the accuracy of the approach, the effect of bulk flow rotation on pressure drop is considered. While the flow rotation and fluid transport from a $y$-direction cell (Fig. 2.b) to an $x$-direction cell (Fig. 2.c) is a complex phenomenon, the 2D projection of similar flow can be considered as a stagnation-point flow.\textsuperscript{15} It has been shown\textsuperscript{13} that the excess pressure drop resulting from bulk rotation can be estimated from the average length of surrounding boundaries. Employing the same approximation here and only considering rotation about $x$-axis, the excess pressure drop resulting from the bulk rotation from a $y$-direction cell to an $x$-direction cell is estimated to be equal to the pressure drop in the $x$-direction cell (the average length of swept boundaries in a rotation is equal to the average length of swept boundaries in the cell, see Ref.\textsuperscript{13}). As the flow in each $x$-direction cell finally rotates again to a new $y$-direction cell, the pressure drop values of the $x$-direction cells are multiplied by 3 to cover these excess pressure drops. While this estimation assumes that the total flow of an $x$-direction cell immediately rotates into a $y$-direction cell, part of the flow can transfer to a neighboring $x$-direction cell and therefore, this simplification of the analysis can result in an overestimation of the pressure drop in the $x$-direction cells. To minimize this effect and retain the simplicity of the approach, the transverse Laplacian term is not considered for $x$-direction cells. As will be shown in the next section (Table 1), the errors of the approach for different samples are not generally skewed towards over- or under-estimations, which supports the idea that the accuracy level of the approach is not completely governed by these approximations.

### 3. Model verification

To assess the accuracy of the model, the flow between two parallel plates having various artificial asperities was modeled using the COMSOL\textsuperscript{TM} multiphysics computational fluid dynamics package and the results are compared with the model predictions. The asperities are made by attaching 1-cm-radius hemi-spheres to 6 cm × 6 cm plates. While the attached hemi-spheres are arranged symmetrically in each plate, they are in a staggered (S), parallel (P) or middle (M) arrangement in comparison to the other plate for different cases (see Fig. 3). The transverse flow is also studied in the parallel (PT) and middle (MT) cases. The flow between a smooth top plate and the asperity-attached bottom plate (D) is also modeled and compared. Fig. 3 shows a sample geometry of the cases in COMSOL\textsuperscript{TM} (Fig. 3.a), different arrangements seen from above (Fig. 3.b-g), and sample 3D geometries (Fig. 3.h-k). The inlet flow and the outlet are positioned 10 cm and 20 cm away from the attached hemi-spheres in COMSOL\textsuperscript{TM}, respectively, to minimize the boundary condition effects. The side boundary conditions are

<table>
<thead>
<tr>
<th>Case</th>
<th>Permeability ($m^2$)</th>
<th>Logarithmic difference</th>
<th>Percentage difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Code</td>
<td>COMSOL\textsuperscript{TM}</td>
<td>Presented model</td>
</tr>
<tr>
<td>1</td>
<td>S1.5</td>
<td>2.64e-6</td>
<td>3.17e-6</td>
</tr>
<tr>
<td>2</td>
<td>S1.1</td>
<td>7.74e-7</td>
<td>6.99e-7</td>
</tr>
<tr>
<td>3</td>
<td>S0.9</td>
<td>3.68e-7</td>
<td>3.10e-7</td>
</tr>
<tr>
<td>4</td>
<td>P1.5</td>
<td>4.60e-6</td>
<td>5.46e-6</td>
</tr>
<tr>
<td>5</td>
<td>P1.1</td>
<td>2.49e-6</td>
<td>2.90e-6</td>
</tr>
<tr>
<td>6</td>
<td>P0.9</td>
<td>1.71e-6</td>
<td>1.96e-6</td>
</tr>
<tr>
<td>7</td>
<td>PT1.1</td>
<td>6.28e-8</td>
<td>5.12e-8</td>
</tr>
<tr>
<td>8</td>
<td>MT1.1</td>
<td>1.67e-7</td>
<td>1.66e-7</td>
</tr>
<tr>
<td>9</td>
<td>M1.1</td>
<td>1.57e-6</td>
<td>1.55e-6</td>
</tr>
<tr>
<td>10</td>
<td>D0.1</td>
<td>3.09e-8</td>
<td>2.52e-8</td>
</tr>
<tr>
<td>11</td>
<td>D0.5</td>
<td>7.28e-7</td>
<td>7.75e-7</td>
</tr>
<tr>
<td>12</td>
<td>D0.9</td>
<td>2.26e-6</td>
<td>2.44e-6</td>
</tr>
</tbody>
</table>
Fig. 3. Cases used for model verification: (a) The sample geometry used in COMSOL™; (b) Staggered (S) arrangement; (c) Parallel (P) arrangement; (d) Middle (M) arrangement; (e) transverse parallel (PT) arrangement; (f) transverse middle (MT) arrangement; (g) Smooth top plate case (D). In subfigures (b–f), the asperities on the bottom plates are shown with brighter colors. (h) 3D geometry of the case S0.9; (i) 3D geometry of the case P0.9; (j) 3D geometry of the case D0.5; (k) 3D geometry of the case M1.1.
considered symmetrical. The pressure drop of fluid along the sample's length is calculated and converted to an estimate of permeability. The model was developed by in-house coding of the approach using MATLAB™. A 100 × 100 mesh size was employed to capture the geometry in the model (see Fig. 3.b–g). Here, each x-direction and y-direction cell (Fig. 2) is made up of a 5 × 5 mesh. Each cell is averaged in the transverse direction and then magnified into a larger cell (with maximum size of 20, depending on the aperture variations in the cell) to estimate the pressure drop of the cell.

The predicted permeability values for different cases using this approach and the corresponding results obtained by using COMSOL™ multiphysics, are shown in Table 1. The case code shows the sample arrangement and the distance between the plates. As an example, code S1.1 specifies the case of a staggered arrangement with a 1.1-cm distance between the top and bottom plates. The 3D geometries of cases S0.9, P0.9, D0.5, and M1.1 are also shown in Fig. 3.h–k for better clarity. Considering the size of the attached asperities (1-cm-radius hemi-spheres), all the cases, other than S1.5, S1.1, P1.5, M1.1 and MT1.1, have contact areas between the top and the bottom plates. The table shows that the permeability of the cases examined covers more than two orders of magnitude (3.09ε-8 m² for case D0.1 up to 4.60ε-6 m² for case P1.5). The presented model can predict the permeability values with a logarithmic difference of less than 0.1 (a percentage difference of less than 20%) compared to COMSOL™ modeling results for all the samples in this range. This difference is most likely result of the assumptions employed in the model and the effects of the sample on the upstream and downstream flow fields in the COMSOL™ modeling, which is not considered in the model presented here. Therefore, the accuracy of the model is expected to be higher than this value. This subject is discussed further in the following sections.

To reproduce the slope of the fracture, the cells on the base of the sample on the upstream and downstream flow fields in the COMSOL™ modeling, which is not considered in the model presented here. Therefore, the accuracy of the model is expected to be higher than this value. This subject is discussed further in the following sections.

The relationship between permeability and the joint roughness coefficient (JRC) of a fracture is studied here as an example of the model applications. First, the reason for choosing JRC as the indicator of the fracture faces is presented and various approaches employed in the literature to reproduce the fracture face are reviewed. The approach developed here to reproduce the fracture faces, which is based on the JRC and fractal dimension, is then discussed. Finally, the JRC-permeability correlation is studied using the presented model and the results are compared with correlations available in the literature.

5. Reproduction of the fracture faces

Several approaches have been employed in the literature to reproduce the fracture face for numerical analysis. The simplest approach is to consider the fracture area as a separate computational domain with a constant aperture. The permeability in this domain can then be estimated using the cubic law.1,2 While the accuracy of this estimation is questionable, it helps researchers focus on other aspects of the fracture response and transport capabilities.

The most accurate approach is to scan a physical fracture and model the extracted geometry and resultant flow based on the real fracture surface data. Various contact and non-contact methods have been used to study and scan the fracture surfaces.10 A 2D6,11 or 3D8,9,12 projection of the fracture face was produced and employed in the modeling process. While the 3D approach results in the most accurate possible investigation of a specific fracture, the generalization of the results is not straightforward. In other words, it is not clear how one can extend the results from such studies to similar fractures in the field, as similarity parameters are not clarified.

To resolve this issue, the roughness of the fracture face has to be quantified and an indicative parameter (or a group of parameters) has to be extracted to represent the fracture face with a one-to-one correspondence. However, while more than 20 parameters have been presented to quantify a fracture face, the surface roughness has not yet been properly quantified.16,18,19 Nevertheless, two different approaches have been employed in the literature to reproduce the fracture faces numerically using a set of parameters. In one approach, the surface is assumed to have a self-affine topography and is reproduced based on the considered fractal dimension.5,10,11,18,20 The self-affinity assumption and the accuracy of estimating the fractal dimension of a physical fracture face has raised concerns in the literature16,18; nevertheless, this approach is commonly employed.

In the other approach, the fracture surface is reproduced based on the assumed roughness parameters (specifically the JRC),21 and the references cited therein. The JRC is an interesting parameter as its measurement both in the laboratory and field environments is quite straightforward (tilt, push or pull test22), and its measured value is consistent if the average of several tests is considered (at least ten tests); it is also independent of normal stress in the range of up to eight orders of magnitude.21 Nevertheless, it is more accurate to assume a range for the JRC value of a rock fracture, rather than a specific number (e.g., Table 2 in26).

Here, a compound method is developed to reproduce 3D fracture surfaces. The JRC is a roughness coefficient in the range of 0–2022 that was introduced to estimate the peak shear stress of a rock. Barton and Choubey17 presented typical roughness profiles for various JRC values. Tse and Cruden23 studied the correlations of these typical profiles with available roughness coefficients presented in the literature and found the strongest correlation between the JRC and Z2 parameters. Z2 is the root mean square of the first derivative of the profile, which can be presented as

\[ Z_2 = \sqrt{\frac{1}{L} \int_0^L \left( \frac{dy}{dx} \right)^2 dx^{1/2}} \]  

where \( L \) is length of the sample and \( \frac{dy}{dx} \) is the local slope of the profile. The JRC-Z2 relationship has been studied extensively in the literature and various equations have been presented.27,28 Here, the correlation presented by Yu and Vayssade29 is employed, which has the smallest measuring interval and high correlation coefficient (0.25 mm²) and has the form

\[ \text{JRC} = 60.32Z_2 - 4.51 \]

Once the JRC and \( Z_2 \) are known, the average of the \( (\frac{dy}{dx})^2 \) term can be determined (\( (\frac{dy}{dx})^2_{\text{avg}} \)). The \( (\frac{dy}{dx})^2 \) distribution cannot be assumed to be a normal distribution as it is always a positive number. Here, an exponential distribution for the \( (\frac{dy}{dx})^2 \) is assumed. The probability density function (PDF) of an exponential distribution with a mean value of \( \beta \) for a random variable \( x \) can be written as

\[ f(x; \beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases} \]  

To reproduce the 3D surface of the fracture, the cells on the base and left sidelines of the surface (see Fig. 5) are considered first. Assuming the JRC value, the average value for \( (\frac{dy}{dx})^2 \) (\( (\frac{dy}{dx})^2_{\text{avg}} \)) can be calculated using Eqs. (5) and (6). A random value for \( (\frac{dy}{dx})^2 \) is obtained using the PDF given by Eq. (7). The elevation of the next cell on the sideline is then determined based on the calculated slope and the elevation of the adjacent cell. When the elevations of sideline cells are specified, elevations of the inner cells are determined accordingly. Considering cell \( x \) in Fig. 5, the elevation is obtained such that the average of \( (\frac{dy}{dx})^2 \) for cells \( a, b, c, d \) and \( x \) becomes equal to \( (\frac{dy}{dx})^2_{\text{avg}} \). The elevation of cell \( x (h_x) \) can be determined as
Here, $h_a$, $h_b$, $h_c$, and $h_d$ are the heights of the cells $a$, $b$, $c$, and $d$, shown in Fig. 5. The above equation can result in two different elevations ($h_x$ values). Similarly, for each sideline cell, two different elevations are obtained in this method as $(dy/dx)^2$ is determined. The elevation is selected based on the self-affinity assumption.

To satisfy self-affinity, the fractal dimension is imposed using the original divider method. According to this method, the measured length of the fractal surface ($L$) and the length of the divider ($L_d$) used as the gauge have the relationship of

$$\log L = (1 - D)\log L_d + C$$

where $D$ is the fractal dimension of the surface and $C$ is the intercept of $\log L - \log L_d$ plot. Here, the 1D fractal dimension of 1.2 (equivalent to the universal Hurst exponent of 0.8) is used. For each sideline cell, the last 30 cells on the sideline are considered and the length of the surface

Table 2
The numerical values of ($e_h/e_m$)$^2$ (permeability ratio of rough to smooth surfaces) for 100 random samples in different JRCs and the aperture size for exactly matched surfaces.

<table>
<thead>
<tr>
<th>JRC</th>
<th>Aperture size (mm)</th>
<th>$e_h/e_m$ Min–Max (Average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>($e_h/e_m$)$^2$</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1</td>
<td>0.997–0.998 (0.998)</td>
<td>0.992–0.994 (0.993)</td>
</tr>
<tr>
<td>3</td>
<td>0.991–0.993 (0.992)</td>
<td>0.986–0.988 (0.987)</td>
</tr>
<tr>
<td>5</td>
<td>0.983–0.985 (0.984)</td>
<td>0.977–0.979 (0.978)</td>
</tr>
<tr>
<td>7</td>
<td>0.972–0.975 (0.974)</td>
<td>0.967–0.969 (0.968)</td>
</tr>
<tr>
<td>9</td>
<td>0.959–0.962 (0.961)</td>
<td>0.954–0.957 (0.956)</td>
</tr>
<tr>
<td>11</td>
<td>0.941–0.945 (0.944)</td>
<td>0.939–0.942 (0.940)</td>
</tr>
<tr>
<td>13</td>
<td>0.919–0.924 (0.922)</td>
<td>0.919–0.923 (0.921)</td>
</tr>
<tr>
<td>15</td>
<td>0.884–0.900 (0.897)</td>
<td>0.882–0.902 (0.899)</td>
</tr>
<tr>
<td>17</td>
<td>0.865–0.872 (0.868)</td>
<td>0.867–0.875 (0.871)</td>
</tr>
<tr>
<td>19</td>
<td>0.830–0.840 (0.835)</td>
<td>0.831–0.845 (0.839)</td>
</tr>
</tbody>
</table>

Fig. 4. Colormaps of velocity distributions in the $y$ (top) and $x$ (bottom) directions predicted using COMSOL™ (left) and the current model (right) for case M1.1.
(L) is determined using divider lengths (l_d) of 1, 2, 5, and 10 cells; the elevation of the cell is selected such that the slope of the log-log graph of the L-l_d relationship becomes closer to (1-D). As there are not enough cells (i.e., 30 cells) behind the first cells in the row, the following procedure is employed:

1. The sign of the slope of the first 6 cells is selected randomly (the absolute value, known as (\(\frac{dy}{dx}\))^2, was obtained previously).
2. For cells 7–10, only the divider lengths of 1 and 2 cells are considered.
3. For cells 11–30, only the divider lengths of 1, 2 and 5 cells are considered.

To minimize the effects of the above algorithm on self-affinity of the surface, the first 60 cells in the row are ignored and the sideline cells of the fracture surface are considered from cell 61 in this row. For the inner cells, on the other hand, the horizontal or vertical row of cells from the sidelines to the cell (whichever is longer) is considered. If the number of cells in this row are greater than 30, all divider lengths are considered. Otherwise, only divider lengths of 1, 2 and 5 for rows with between 11 and 30 cells, and only divider lengths of 1 and 2 for rows with less than 10 cells, are considered. Since the horizontal and vertical rows of cells are longer on two opposite sides of the diagonal of the surface, the asperities might show different directional preferences on these sides (e.g., see two sides of the vertical diagonal in Fig. 6). However, the fractal and other analysis presented below prove that the results are not affected by this.

Fig. 6 depicts two sample surfaces reproduced by this method with JRC values of 1 and 10. As expected, the higher JRC value results in higher asperity values and deviations. While the fractal dimension of 1.2 is imposed on the surface, the elevations of the cells are also controlled by JRC values, and the final fractal dimensions are different, varying with the JRC. Fig. 7 shows the measured fractal dimensions in 11 horizontal and 11 vertical lines of a sample for each JRC value. The figure shows that the fractal dimension of a surface is generally consistent in different positions and directions, specifically for lower JRC values. Several relationships have been presented in the literature to relate the JRC and fractal dimension, and the agreement with these results is satisfactory. This consistency is especially interesting here as it suggests that the reproduced surfaces can be considered to be a reliable representative of a JRC value.

6. Correlation between JRC and permeability

The limitations of the cubic law for rough surfaces have been extensively discussed in the literature e.g.,. As mentioned before, various parameters have been presented to quantify the roughness of the surface and the JRC is specifically notable for practical purposes. It can be measured easily both in the laboratory and at field scales and its value is consistent and independent of the applied normal stress over a wide range. Based on the cubic law (Eq. (1)), permeability of a fracture can be determined as

\[
k = \frac{e_m^2}{12}
\]

where \(e_m\) is the aperture height and is called the mechanical aperture. It is assumed that for a rough surface, the above relationship is still valid if the mechanical aperture (\(e_m\)) is replaced by a hydraulic aperture (\(e_h\)) that covers the roughness effects. In this case, the permeability of a rough fracture is determined as follows

\[
k = \frac{e_h^2}{12}
\]

There is an extensive body of literature on the relationship between the hydraulic aperture and roughness parameters of a surface e.g.,. For the reasons mentioned before, only the JRC parameter is reviewed here as the roughness parameter. Barton et al. presented the most famous empirical relationship based on an approximate evaluation of presented experimental data in the literature, which has the form of

\[
e_h = \frac{e_m^2}{\text{JRC}^{2.5}}
\]
While this correlation is not dimensionally appropriate, a more significant concern arises when Eqs. (11) and (12) are combined. This combination predicts that permeability is correlated to \( c_f^2 \) for a rough surface (Eqs. (11) and (12)) and \( c_s^2 \) for a smooth surface (Eq. (10)); this does not seem reasonable, especially when it is recalled that the effects of roughness disappear for larger apertures.

The other relationships\( ^7, ^34 \) are essentially the result of 2D modeling of flow between typical profile visualizations given by Barton and Choubey.\( ^23 \) Rasouli and Hosseinian\( ^35 \) assumed different JRC values for the two faces of the fracture and employed the average JRC value in their correlation. This assumption specifically requires revisitation since the JRC values of the faces of a real fracture are essentially equal. Zoorabadi et al.\( ^7 \) studied the flow between the matched surfaces and verified their modeling estimations by measuring the permeability of the flow between the 2D profiles presented by Barton and Choubey.\( ^23 \) Crandall et al.\( ^6 \) studied 3D flow in a fracture and the JRC-transmissivity relationship. They performed a micro-CT scan on a fracture created using a modified Brazilian technique and then developed different meshes (with different JRC values) using various meshing techniques on the same fracture. Since different meshes essentially resulted from the same fracture, the meshes (with different JRC values) might not be an accurate representation of real surfaces with different JRC values.

Here, the correlation between the JRC and permeability was evaluated statistically using 3D fracture surfaces reproduced by the algorithm explained in the previous section. In each case, a 100 × 100 mesh size was employed to model the fracture. The \( x \)-direction and \( y \)-direction cells (Fig. 2) of a 5 × 5 mesh size were considered, which were then magnified into larger cells with a maximum size of 20 (similar to the explanation of the verification process). A cell size of 0.25 mm was selected (equal to the measuring interval of the employed JRC-0.25 correlation, Eq. (6)). For small apertures (when the distance between top and bottom surfaces in the magnified cell is less than one cell), the cell size is halved successively. As an example, for the aperture size of 0.1 mm, the cell size is decreased to 0.0625 mm, while the \( Z_0 \) intervals remain unchanged (0.25 mm). As the overall size of the sample in this case decreases to 1/16th of its original size (from 100 × 100 × (0.25)\(^2 \) mm\(^2 \) to 100 × 100 × (0.0625)\(^2 \) mm\(^2 \)), an average of 16 samples is reported in each case in order to cancel the effects of the sample size. This sample size effect is discussed in detail in the next sections.

Fig. 8 represents the correlation between the JRC and \( (c_d/c_m)^2 \) in different aperture sizes for exactly matched upper and lower surfaces. The \( (c_d/c_m)^2 \) parameter is used because it is equivalent to the ratio of the permeability of the rough surfaces to the smooth surfaces. For each case, 100 realizations of different random surfaces were reproduced and the minimum, maximum, and average values are reported in Table 2; the results presented by Zoorabadi et al.\( ^7 \) are also given.

Several points should be discussed in regard to the results presented in Fig. 8 and Table 2. While 100 different visualizations were used in each case, the permeability range was narrow in all the cases. The resulting values are generally in agreement with those presented by Zoorabadi et al.\( ^7 \), which were verified against their measurements. This observation demonstrates the accuracy of our model. Also, Zoorabadi et al.\( ^7 \) employed the exact 2D profiles reported by Barton and Choubey\( ^23 \); the agreement between their results and the results reported here indicates that the algorithm employed here to reproduce 3D surfaces generally results in 3D visualizations of the 2D standard profiles of Barton and Choubey.\( ^23 \) Finally, these results show that the roughness effect on permeability decrease is less than 20% for exactly matched surfaces even in the worst cases studied (highest JRCs, lowest asperities).

6.1. Translated surfaces

Exactly matched surfaces have always been considered previously in the published literature; however, the reality under field and experimental conditions can be different. Fractures without gouge normally occur due to various types of tensile stress fields, so it is reasonable to expect that the surfaces of the fractures experience relative displacement (e.g., translation) and the exactly-matched-surfaces assumption can be misleading.\( ^35, ^36 \) Even in laboratory measurements, an opened fracture is difficult to replicate exactly. While gouge production and rotation can also result in unmatched surfaces, only translation is discussed here. Fig. 9 shows the effects of rigid-body translation of the surface in the \( y \)-direction (Fig. 1.a) in different JRCs for an aperture size of 0.1 mm. In each case, 100 different samples for JRC values of 1, 5, 10, 15 and 20 were studied and the range of the calculated \( (c_d/c_m)^2 \) values are reported. The results are presented in three different plots for clarity. An aperture size of 0.1 mm was selected since this has been reported in the literature as a nominal aperture size e.g.,\( ^3, ^5, ^7 \).

Fig. 9 depicts that even a relative translation of 0.0625 mm (62.5 \( \mu \)m) has a noticeable effect on the permeability of a fracture. The permeability constantly decreases up to a 0.25-mm translation, and can be one order of magnitude smaller for high JRC values. The permeability begins to increase first at the high JRCs (0.5-mm translation, Fig. 9.a) and, when the translation reaches to 4 mm or higher, the permeability increase is seen for all values of the JRC (Fig. 9.b and .c). Interestingly, the ratio of hydraulic to mechanical apertures shifts to values greater than one for higher translations. In other words, the flow concentrates in local openings and the overall pressure drop becomes lower than the cubic law assumption. The results show that in the case of a 16-mm translation, for all the cases studied with JRCs greater than 5, the permeability ratio is greater than one. To estimate how much translation is required to achieve a reasonable accuracy for the assumption of having uncorrelated surfaces, two independently generated surfaces were considered as the top and bottom surfaces and the permeability ratio calculated. Fig. 9.c shows that even after an 8-mm translation, the surfaces are hydraulically correlated and thus cannot be considered independent. However, if the translation is increased to 16 mm, the range of the permeability ratio with different JRC values becomes reasonably close to the independent surfaces.

In order to understand the effect of aperture size on translation, the modeling was repeated for an aperture size of 0.05 mm. Fig. 10 shows the results for 100 different samples for translations of 0.125 mm, 0.25 mm, 1 mm, 2 mm, 4 mm and 8 mm with a JRC value of 10 for aperture sizes of 0.05 mm and 0.1 mm. The figure shows that the range of permeability variation for the lower aperture size (0.05 mm) is considerably wider. Also, the surfaces can be considered hydraulically independent for the lower translations when the aperture size is smaller. As the results demonstrate, the permeability range is close enough to the independent surfaces for translations of 8 mm at an
aperture size of 0.05 mm, while the results for an aperture size of 0.1 mm do not converge to the results for the independent surfaces. As mentioned before, this independency can be obtained for an aperture size of 0.1 mm after 16 mm translation.

6.2. Independent surfaces

The correlation between the JRC and permeability ratio \( (e_I/e_m)^2 \) at different aperture sizes for independent surfaces are shown in Fig. 11 and Table 3. As the figure shows, the correlation is substantially more significant for the exactly matched surfaces (Fig. 8). Although the accuracy of the results reported (especially the ranges of permeabilities) can be affected by the sample size (100 samples for each case), the following observations are relevant:

1. For large apertures, the cubic law can predict the average permeability with an acceptable accuracy; however, in specific cases (especially in high JRCs) it can give up to one order of magnitude error.
2. For small aperture sizes and moderate to high JRCs, the cubic law underestimates the permeability up to 2 orders of magnitude.
3. The JRC effect is significantly controlled by the aperture size. With high JRCs, the change in aperture size can increase the permeability ratio (ratio of the actual permeability to the cubic law estimation).

![Fig. 9](image9.png) The effect of translation of the surfaces for different JRC values (aperture size of 0.1 mm); (a) translation to aperture ratio of 0–5; (b) translation to aperture ratio of 10–40; (c) translation to aperture ratio of 80–160.

![Fig. 10](image10.png) The effect of aperture size in different translations for JRC value of 10.

![Fig. 11](image11.png) Correlation between the JRC and permeability ratio for different aperture sizes with independent surfaces (see also Table 3).

![Table 3](table3.png)
two typical samples, sizes of 2.5 × 2.5 cm$^2$ and 25 × 25 cm$^2$, with an aperture size in experimental measurements. Fig. 12 depicts the histogram of the aperture size distributions (apparent heights) of one side of aperture sizes and higher JRC values (results are not reported here). It is assumed that the permeability of a larger sample can be estimated from the average of the permeability values of its smaller samples. While this assumption is not in general accurate and permeability of a larger sample can even be affected by the spatial distribution of permeabilities of smaller sections, it is generally accepted that it can represent a reasonable estimation of the larger sample. This assumption becomes questionable when outlier permeabilities are available in smaller sections.

Considering the uncertainties that result from the above assumptions, 1000 different visualizations for JRC values of 1, 5, 10, 15 and 20 and aperture size of 0.1 mm were produced and the permeability ratios calculated. Using the central limit theorem, the standard deviation of the permeability ratio for different sample sizes was calculated and is given in Table 4.

The sample size can also affect the accuracy of estimating the aperture size in experimental measurements. Fig. 12 depicts the histograms of the aperture size distributions (apparent heights) of one side of two typical samples, sizes of 2.5 × 2.5 cm$^2$ and 25 × 25 cm$^2$, with an aperture size of 0.1 mm and a JRC value of 10. The figure shows that the error resulting from the aperture size estimation is considerably increased for larger sample sizes. This issue is more apparent for smaller aperture sizes and higher JRC values (results are not reported here). It should be noted that a two-fold error in aperture size estimation leads to one order of magnitude error in permeability evaluation.

<table>
<thead>
<tr>
<th>JRC</th>
<th>Aperture size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.18-3.22 (2.39)</td>
</tr>
<tr>
<td>0.2</td>
<td>6.92-1.85 (1.19)</td>
</tr>
<tr>
<td>0.5</td>
<td>5.96-1.60 (1.01)</td>
</tr>
<tr>
<td>1</td>
<td>0.776-1.27 (0.998)</td>
</tr>
</tbody>
</table>

Table 3: The numerical values of permeability ratios for 100 samples with different JRCs and aperture sizes for independent surfaces (see also Fig. 11).

Table 4: The average and standard deviations of permeability ratios for 1000 samples with an aperture size of 0.1 mm and independent surfaces.

<table>
<thead>
<tr>
<th>JRC</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.24</td>
<td>0.233</td>
</tr>
<tr>
<td>5</td>
<td>2.36</td>
<td>0.576</td>
</tr>
<tr>
<td>10</td>
<td>4.83</td>
<td>1.25</td>
</tr>
<tr>
<td>15</td>
<td>9.13</td>
<td>2.72</td>
</tr>
<tr>
<td>20</td>
<td>14.7</td>
<td>4.29</td>
</tr>
</tbody>
</table>

6.3. Sample size effect

There have been many reports in the literature that fracture properties are influenced by scale effects e.g.,. Here, the effect of sample size on the permeability of the fracture is studied. Two important assumptions are employed in this analysis:

1. The JRC values are assumed to remain constant for different sample sizes. While this assumption is not supported by experimental studies, there is not enough theoretical background to explain this phenomenon. Specifically, if the JRC-Z$_2$ relationships, such as the one presented in Eq. (6), are assumed to be correct, it is not clear how the Z$_2$ parameter is affected by the sample size. It has to be remembered that the divider length does not change when the sample size changes.

2. It is assumed that the permeability of a larger sample can be estimated from the average of the permeability values of its smaller regions (e.g., permeability of a 1-m$^2$ fracture is the average of four 0.5-m$^2$ fractures). While this assumption is not in general accurate and permeability of a larger sample can even be affected by the spatial distribution of permeabilities of smaller sections, it is generally accepted that it can represent a reasonable estimation of the larger sample. This assumption becomes questionable when outlier permeabilities are available in smaller sections.

Considering the uncertainties that result from the above assumptions, 1000 different visualizations for JRC values of 1, 5, 10, 15 and 20 and aperture size of 0.1 mm were produced and the permeability ratios calculated. Using the central limit theorem, the standard deviation of the permeability ratio for different sample sizes was calculated and is given in Table 4.

The sample size can also affect the accuracy of estimating the aperture size in experimental measurements. Fig. 12 depicts the histograms of the aperture size distributions (apparent heights) of one side of two typical samples, sizes of 2.5 × 2.5 cm$^2$ and 25 × 25 cm$^2$, with an aperture size of 0.1 mm and a JRC value of 10. The figure shows that the error resulting from the aperture size estimation is considerably increased for larger sample sizes. This issue is more apparent for smaller aperture sizes and higher JRC values (results are not reported here). It should be noted that a two-fold error in aperture size estimation leads to one order of magnitude error in permeability evaluation.

6.4. Contact area

The aperture size and JRC value of a fracture also control the ratio of the contact area between the surfaces of the fracture. The ratio of contact area to the overall area is especially important as it is correlated to the aperture size variations resulting from normal stress and also the fluid flow transport in the fracture. While the distribution of asperity height is as significant as the contract area ratio in the permeability changes due to normal stress, stress-displacement modeling is required to properly correlate the contact area ratio to the normal stress. Table 5 gives the contact area distribution for different aperture sizes and JRC values. The values presented in the table are in agreement with the values reported before in the literature for high aperture sizes and low JRCs, the contact area percentages for other cases are all skewed towards lower contact area percentages (medians are always smaller than averages).

The limited contact area percentages even in low aperture sizes (i.e. high normal stresses) supports the assumption that the JRC value range of a fracture does not change in a wide range of applied stresses. While a certain amount of in-contact asperities are assumed to be high normal stress, since the contact area percentage remains less than 10% on average, the local slope of the profile (Z$_2$) (Eq. (5)) and so the JRC value (Eq. (6)) vary in the same range (i.e., less than 10%) at most. Considering the available uncertainty in determining the JRC value, the variations of the JRC value range as a result of normal stress can therefore be safely neglected.

7. Conclusions

A new modeling approach for the fluid transport in fractures is presented. Different categories of modeling and their limitations were discussed and the corrections applied in the presented model compared to models available in the literature were explained. The accuracy of the model was then analyzed by comparing the model predictions with the results obtained using the COMSOL package for different geometries. A new method for reproducing the fracture surfaces was presented.

Table 5: The contact area percentage for different categories of modeling and their limitations were compared.
based on both the JRC and fractal dimension. The fractal dimension of a constructed surface was measured and compared with reported values in the literature. The model was then used to study the JRC-permeability correlation. For each case, 100 different visualizations were reproduced and the range of calculated permeabilities was recorded. For exactly matched surfaces, the results were consistent with the results reported in the literature. Translated surfaces were then studied and it was shown that even a translation of 62.5 µm for an aperture size of 0.1 mm has a noticeable effect on the permeability range. It was demonstrated that as translation increases, the permeability first decreases and then increases to values greater than the cubic law estimations. The shifting threshold beyond which surfaces can be considered independent was estimated to be 16 mm and 8 mm for aperture sizes of 0.1 mm and 0.05 mm, respectively (i.e., translation of 160 times the aperture size). The JRC-permeability correlation for the case of independent surfaces was then analyzed and it was shown that for large aperture sizes, the cubic law can predict the permeability with a reasonable accuracy (although in special cases it can have up to one order of magnitude error), while for small aperture sizes and moderate to high JRC values, the cubic law underestimates the permeability up to 2 orders of magnitude. The sample size effect and the assumptions employed were discussed and it was shown that the aperture estimation error could be considerably increased for larger sample sizes. Finally, the contact area ratios for different JRCs and aperture sizes were presented.

Table 5
Contact area percentages of 100 samples with different aperture sizes and JRC values for independent surfaces.

<table>
<thead>
<tr>
<th>JRC</th>
<th>Aperture size (mm)</th>
<th>0.05</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contact Area Percentage (%)</td>
<td>Average, Median, Skewness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.17, 0.32, 3.46</td>
<td>0.208, 0, 4.53</td>
<td>0, 0, –</td>
<td>0, 0, –</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5.18, 3.44, 1.76</td>
<td>1.89, 0.62, 3.02</td>
<td>0, 0, –</td>
<td>0, 0, –</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6.99, 4.16, 3.04</td>
<td>3.51, 2.07, 2.85</td>
<td>0.0481, 0, 5.93</td>
<td>0, 0, –</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>9.64, 6.34, 2.87</td>
<td>4.60, 3, 1.97</td>
<td>0.371, 0.09, 4.06</td>
<td>0, 0, –</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>7.13, 4.71, 3.59</td>
<td>6.39, 3.92, 2.97</td>
<td>1.29, 0.29, 4.23</td>
<td>0.0092, 0, 8.21</td>
<td></td>
</tr>
</tbody>
</table>

considered independent was estimated to be 16 mm and 8 mm for aperture sizes of 0.1 mm and 0.05 mm, respectively (i.e., translation of 160 times the aperture size). The JRC-permeability correlation for the case of independent surfaces was then analyzed and it was shown that for large aperture sizes, the cubic law can predict the permeability with a reasonable accuracy (although in special cases it can have up to one order of magnitude error), while for small aperture sizes and moderate to high JRC values, the cubic law underestimates the permeability up to 2 orders of magnitude. The sample size effect and the assumptions employed were discussed and it was shown that the aperture estimation error could be considerably increased for larger sample sizes. Finally, the contact area ratios for different JRCs and aperture sizes were presented.

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References


