Some experimental studies concerning the contact stresses beneath interfering rigid strip foundations resting on a granular stratum

A. P. S. SELVADURAI
Department of Civil Engineering, Carleton University, Ottawa, Ont., Canada K1S 5B6
AND
S. A. A. RABBAA
Department of Civil Engineering, Al-Azhar University, Madinet Nasr, Cairo, Egypt
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This paper presents an experimental study of the contact stress distribution beneath two interfering rigid strip foundations resting in frictionless contact with a layer of dense sand underlain by a smooth rigid base. It is found that the interference between the two foundations has a significant influence on the contact stress distribution. In the absence of interference, the contact stress distribution beneath a single foundation exhibits a symmetrical shape. As the spacing between the foundations diminishes the contact stress distribution exhibits an asymmetrical shape.

Keywords: contact stresses, foundation interference, plane strain tests, experimental studies.

Introduction

The study of the interaction between structural foundations and the supporting soil media is of fundamental importance to both geotechnical and structural engineering. Information regarding contact stress distribution, settlement, and bearing capacity is required for an adequate design of the foundation. The mutual interference of foundations (Fig. 1) in a group has a significant influence on these design factors.

The influence of group behaviour on the load carrying capacity of footings is examined in the articles by Stuart and Hanna (1961), Stuart (1962), Myslivec and Kysela (1963), Mandel (1963, 1965), West and Stuart (1965), Murthy (1970), Singh et al. (1973), Myslivec and Kysela (1973), Swami and Agarwal (1974), Siva Reddy and Mogaliah (1976), Khadilkar and Varma (1977) and Deshmukh (1978). The investigations by Murthy (1970), Singh et al. (1973), and Khadilkar and Varma (1977) place some emphasis on the influence of the interference on the settlement of the footings. To date (see, e.g., Desai and Christian 1977; Meyerhof 1979; Selvadurai 1979, 1983) the influence of the interference between foundations on the contact stress distribution in particular appears to have received little or no attention.

This paper is primarily concerned with an experimental study related to the measurement of the contact stress distribution beneath two interfering rigid strip foundations resting in smooth contact with a layer of sand which in turn is underlain by a smooth rigid base. The contact stresses beneath a foundation form a vital part of the information necessary to evaluate flexural moments and shear forces that are induced in the footing. Conventional geotechnical and structural design guidelines (see, e.g., Fintel 1974; Winterkorn and Fang 1975) assume that the contact stress distribution beneath a centrally loaded strip foundation is uniform. It is generally recognized that this approach is a highly simplified idealization of an extremely complex problem in soil–foundation interaction which is influenced by a variety of factors including (i) the yielding behaviour of the soil, (ii) the relative flexibility of the soil–foundation system, and (iii) the magnitude of the applied load in relation to the maximum or collapse load. In the absence of general theoretical or numerical guidelines which take into account the above soil–foundation effects, the simplified design considerations are adopted. According to these simplified concepts, when the footing is centrally loaded both single or interfering foundations would exhibit the same uniform contact stress distribution (Fig. 2a). There is no account made for the possible nonuniformity in the contact stress distribution that can exist due to the soil–foundation interaction processes in closely spaced foundations (Fig. 2b).
The experimental results presented in the paper discuss the development of asymmetry in the contact stresses at the soil—foundation interface due to the variation in the spacing between the two foundations. The experiment conducted in this study is somewhat hypothetical. It is intended to model a probable "bound" encountered in the description of soil—foundation interaction problems. These bounds are directly related to interface characteristics either between the footing and the soil or the soil and the underlying rigid base. These interfaces can exhibit either frictional (Coulomb-type) or frictionless conditions with finite friction occupying an intermediate position. In a practical situation the nature of the interface between the granular soil layer and an underlying rigid base can rarely be prescribed with certainty. For this reason it is prudent to examine probable variations in the interface behaviour by assigning suitable artificial bounds. In this paper attention is restricted to the smooth idealization. Such a situation can occur at the base of a granular fill or deposit due to accumulation of a layer of fine particles or due to the presence of a thin band of weak soil.

**Experimental program**

The problem modelled in the experimental investigation is shown in Fig. 3. It examines the interference between two rigid strip foundations resting on the surface of a layer of sand underlain by a rigid base. The two foundations are of equal width and are subjected to central loads of equal magnitude. The vertical plane $M_1M_2M_3M_4$ between the two foundations therefore experiences

1. zero displacement in the $x$-direction and
2. exhibits zero shear stresses $\sigma_{xz}$ and $\sigma_{yz}$.

For the purpose of experiments, the plane $M_1M_2M_3M_4$ can be replaced by a rigid immovable plane which has a smooth surface. The planes $C_1(z = +L/2)$ and $C_2(z = -L/2)$ experience

1. zero displacement in the $x$-direction and
2. exhibit zero shear stresses $\sigma_{xz}$ and $\sigma_{yz}$.

Therefore to model the plane strain behaviour it is necessary to provide rigid immovable smooth planes on the surfaces $C_1$ and $C_2$.

From the above discussion it is evident that the symmetrically loaded interfering foundations can be modelled as a single footing problem provided the requisite frictionless and displacement constraints can be accommodated at the soil—test container boundary. The conditions relating to zero displacements (at the plane of footing symmetry and in the plane strain direction) can be achieved by providing a test container which experiences little or no lateral deflections under the pressures induced by the footing (Fig. 4). The second requirement relating to smooth behaviour at the appropriate boundaries is a condition which is much
more difficult to achieve in the context of an experimental simulation. A smooth interface is a mathematical idealization which does not exist in practice (see, e.g., Bowden and Tabor 1964; Ling 1973). In an experimental program it is only possible to approach, in a limited way, the characteristics of smooth interface.

In the experiments performed, the smooth behaviour was modelled by the provision of highly polished No. 16 gauge stainless steel surfaces. The angle of friction for this stainless steel was found to vary between 0° to 7° for normal stresses in the range 0 to 150 kPa. It is possible that boundaries with small amounts of friction can contribute to the development of contact shear stresses at the soil-container interface. However, these influences are bound to be less significant than those which can be encountered in the extreme situation in which the interfaces would exhibit friction angles which approach
the angle of internal friction (plane strain, triaxial or otherwise) of the granular soil. For this paper it will be assumed that the term frictionless interface (or boundary) refers to a highly polished stainless steel interface (or boundary) with an angle of friction which can vary between $0^\circ$ to $7^\circ$. It must be pointed out that any experimental procedure which does not make use of the advantages of the symmetry properties discussed previously will have to contend with large scale continuous or articulated test footings with simultaneous application of the footing loads. The experimental procedures involved in the testing of a full plane strain model of interfering foundations (that is supposed to eliminate boundary friction) are far more complicated and are certainly not without drawbacks.

The inside dimensions of the test tank were approximately $1.52 \times 0.38 \times 0.40 \text{ m}$ (see Fig. 4). A square steel plate of dimensions $378 \times 378 \times 51 \text{ mm}$ and with a smooth bottom surface was used as a model for a smooth rigid strip foundation. Twelve pressure transducers were used to measure the contact stresses beneath the foundation model; six were placed along the section A-A through the centre line and three along each of the sections B-B and C-C close to the edges (see Fig. 5). The test foundation was developed for use in previous studies (see, e.g., Kempthorne 1978; Selvadurai and Kempthorne 1980). Complete details of the steel plate and the pressure transducers are given in the above articles.

The vertical displacements of the test plate were measured by means of four dial gauges (with an accuracy of $0.025 \text{ mm}$) located at each corner of the plate. The four readings indicated the uniformity of plate settlement. The settlement of the plate was taken as the average of the four readings. Surface displacements on either side of the footing were measured by means of a number of displacement transducers (LVDTs) and dial gauges.

The test plate was loaded by a manually operated pump–hydraulic jack system. The hydraulic jack was connected to a steel shaft $38 \text{ mm}$ in diameter. The steel shaft was attached rigidly to a $45 \text{ kN}$ capacity load cell by a threaded connection. This connection was designed to ensure uniform settlement of the test plate.

The granular material used in this study was Ottawa sand, which has the grading curve shown in Fig. 6. The coefficient of uniformity of the soil was 1.7 and the minimum and maximum densities were approximately 15.3 and $18.0 \text{ kN/m}^3$ respectively. The in situ density that could be attained by raining of the sand was $17.3 \text{ kN/m}^3$, which corresponds to a relative density ($D_r$) of $90 \pm 2\%$. For reference it may be noted that the angle of internal friction $\phi$ as obtained from triaxial compression tests on samples with a relative density of $90\%$ was approximately $41^\circ$. In the absence of plane strain test data, this value of $\phi$ may be used to estimate a value for the ultimate bearing capacity of the footing ($q_u$). It is well recognized (see, e.g., Bishop 1966; Roscoe 1970) that the value of $\phi$ is higher in the plane strain test than in a triaxial test. In the context of the present paper, this difference will result in an underestimation of the computed value of $q_u$. Since the experimental programme primarily concentrates on the behaviour of the footing in the working load range, this distinction between $\phi$-triaxial and $\phi$-plane strain is expected to be of marginal interest.
The spacing between the foundation model and its symmetrical counterpart was changed four times using $S/B$ ratios of $1, 2, 3,$ and $4,$ where $S$ is the centre to centre spacing and $B$ is the width of one of the two foundations (see Fig. 3). The depth of the soil layer was fixed at $H = B$. This fixed value of $H$ places a restriction on the applicability of the results to other $H/B$ ratios. Nevertheless, the trends observed in the contact stress measurements should provide a guide to the type of contact stresses that may be encountered for other $H/B$ ratios. Furthermore, the soil deformations which have a significant influence on the contact stress distribution at working loads (in contrast to contact stresses at or near priming load, it is possible to eliminate the mismatch in the footing consistent with its dimensions and the performed to ascertain the maximum bearing capacity of the footing absolute parallel to the surface of the rotation restrained indenting footing. If there are any irregularities at the surface of the layer of sand, they tend to affect the contact stresses that are observed in the first loading cycle. By subjecting the footing to a nominal priming load, it is possible to eliminate the mismatch in the orientation of the footing and surface of the sand layer. Obviously some irreversible deformations are necessary to achieve this compatibility. In the first loading cycle these irreversible deformations would largely be restricted to a zone near the surface of the granular soil layer which possesses little or no strength. Some densification or precompression of the soil layer results in the first cycle of loading. The second and subsequent loading cycles essentially induce pseudoplastic behaviour in the granular layer. This is clearly illustrated by repeatability in the shape of the contact stress profiles for second and subsequent cycles, up to the fourth cycle of loading (see Fig. 7). In considering the uncertainty associated with the bedding errors encountered in the first cycle it is desirable to focus attention on the results obtained for the second and subsequent cycles of loading. The reliable value of the contact stress measured by a certain pressure transducer was taken as the average of the three readings of the second, third, and fourth cycles of loading.

Further details of the material properties, the apparatus, and the test procedures associated with the investigation are also given by Rabbiaa (1981).

**Experimental results**

The load–settlement curves of the plate determined for four different spacing ratios of $S/B = 1, 2, 3,$ and $4$ are shown in Fig. 8. It is noted that the settlement within the range of the maximum load applied $(q_u/3)$ decreases as the spacing decreases. Also, as the spacing increases, the influence of the interference decreases. It can also be noted that the initial part of the load–settlement curve exhibits a concave shape. This concave shape is in contrast to the convex shape that is normally observed in the load–settlement response of a rigid strip footing resting on a soil medium of large depth (see Fig. 9). This difference may be attributed to the effect of the rigid base (and the plane strain conditions) which causes the confining pressure to increase in the region between the two foundations as the load increases. Consequently, the initial tangent modulus of an element in that region increases with increasing load, giving rise to the concave shape. The concave load–settlement curve exists only for a small level of loading; upon further increase in the load, the slope of the load–displacement curve becomes constant (this constant slope region exists up to about one half the computed ultimate load; see Fig. 9). Since the computed ultimate load is beyond the maximum load capacity of the pump–hydraulic jack system used for applying loads, it was not possible to increase the load until failure. A further increase in load is expected to cause a decrease in the slope of the load–displacement curve and failure of the soil medium.

The contact stress distributions corresponding to the spacing ratios $S/B = 1, 2, 3,$ and $4$ are shown in Fig. 10a–d. Each figure shows three diagrams of the contact stress distributions produced at different locations beneath one of the two interfering foundations. The first
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Range of 6 tests

FIG. 8. Load-settlement curves of two interfering rigid strip foundations resting on a layer of sand and underlain by a rigid base.

distribution was obtained by the six pressure transducers located along the centre line of the test plate (section A-A, Fig. 5). The second and the third distributions were obtained by the sets of three transducers located near each edge of the plate (sections B-B and C-C, Fig. 5). If ideal plane strain conditions existed then the three distributions along sections a-a, b-b, and c-c (see Fig. 10a–d) would be the same. It is evident that although the actual magnitudes of the contact stresses exhibit variations, the general trends observed at locations of axes a-a, b-b, and c-c are quite similar. The average values of the contact stress distributions along the two sections near the edges were about 5% lower than that value along the centre line. These differences could be attributed to either some lateral movement of the side walls of the test tank, or the variation of the structure and the density of the deposited sand near the edges. The first possible cause was almost eliminated by restraining the side walls against the steel frame by means of screw jacks (see Fig. 4a, b). Therefore, the edge effect would seem to be the most plausible reason for the discrepancy between the measured contact stresses at the central and edge regions. The average contact stress along the section a-a ($q_0$) was considered to be the most reliable result. The measured contact stress ($q_0$) was normalized with reference to $q_0$.

The contact stress distributions exhibited a convex shape with lower values observed near the free edge (located remote from the symmetric counterpart). This type of distribution differs from the concave shape proposed by Schultze (1961) for a deep stratum subjected to the same level of loading ($q_u/3$). The contact stress distributions immediately under the edges of model footing could not be measured since the pressure cells were offset from the edge (see Fig. 5). The exact value of the contact stress distribution of the edge is open to interpretation and conjecture. The contact stress distribution at the edge of a smooth footing resting on the surface of a granular soil is theoretically zero (Taylor 1948). Although the actual intensity of the stress is zero, the gradient of the contact stress distribution is expected to change quite markedly with the density of the granular soil and the width of the footing. As the density of the soil increases the increase in soil strength is expected to provide a higher contact stress gradient near the edge of the footing (Fig. 11). For this reason, the probable contact stress distributions are left unspecified and indicated by dotted lines (see Fig. 10a–d). The contact stress distributions measured in this series of experiments, however, follow the same pattern as that which was observed by Bauer et al. (1979) for a strip foundation resting on the surface of crushed quartz sand.
Fig. 10. Contact stress distribution beneath one of two interfering rigid strip foundations. (a) Spacing ratio $S/B = 1$. (b) Spacing ratio $S/B = 2$. (c) Spacing ratio $S/B = 3$. (d) Spacing ratio $S/B = 4$. 
Referring to Fig. 10a–d, it appears that the shape of the contact stress distribution depends on the spacing between the two interfering foundations; it varies from an asymmetric shape for a spacing ratio \( S/B = 1 \) to a symmetrical shape for a spacing ratio \( S/B = 4 \). When the spacing ratio is equal to 1, the test plate essentially acts as half of a foundation of width \( 2B \). The contact stress distribution measured under the test plate represents, in this case, half of the complete distribution that would be measured for a width \( 2B \). Hence, the measured contact stress distribution is asymmetrical. The test footing located at the centre of the tank corresponds to a spacing ratio \( S/B = 4 \). In this position, the foundation comes under the influence of two further adjacent foundations (see Fig. 12). As a consequence the contact stress distribution would exhibit a symmetrical shape.

Figure 13 shows the average contact stress distributions corresponding to the spacing ratios from 1 to 4. These distributions were based on the calculated average value of the contact stresses along the sections: a-a, b-b, and c-c shown in Fig. 10a–d. It can be said that the measured contact stress distributions are produced under the combined effect of the concentrated load \( P \) applied at the centre of the plate and an external moment which results from preventing the plate from tilting or rotating. It is convenient to replace this system of a central load \( P \), and the external moment \( M \), by the equivalents \( P \) and an eccentricity \( e \) (where \( e = M/P \)) (see Fig. 14). The eccentricity of the contact stress distribution corresponding to each spacing ratio was computed by using the experimental results shown in Fig. 13. The relationship between the normalized eccentricity \( e/B \) and the spacing ratio \( S/B \) is shown in Fig. 14. This relationship can be described by the following equation, which was obtained by using a least-squares fit:

\[
\frac{e}{B} = 0.1 - 0.025 \frac{S}{B}; \quad 1 \leq S/B \leq 4
\]

An approximate linear distribution of the contact stresses beneath one of two interfering rigid strip foundations, resting on a layer of sand of limited thickness and located at spacing \( S \) from each other, could be obtained as follows:

\[
\begin{align*}
\frac{q_1}{q_2} &= q_a \left[ 1 \pm \frac{6e}{B} \right] \\
\text{and from [1]}:
\frac{q_1}{q_2} &= q_a \left[ 1 \pm \left( 0.6 - 0.15 \frac{S}{B} \right) \right]
\end{align*}
\]

where \( q_1 \) = the contact stress beneath the confined edge, close to the symmetrical counterpart; \( q_2 \) = the contact stress beneath the free edge, far from the symmetrical counterpart; and \( q_a \) = average applied stress on a single footing.

Similar observations concerning the eccentricity and inclination of the soil reaction due to the interference between two rigid strip foundations were reported by West and Stuart (1965).

Concluding remarks

This paper presents an experimental study of the problem of interference between two closely spaced rigid strip foundations resting on a layer of granular soil. The experimental study of a complete model which takes into account the interference involves a considerable amount of experimental effort. The study can,
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SPACING RATIO (S/B)

FIG. 14. Relationship between spacing and eccentricity due to interference of two rigid strip foundations.

(iv) provisions for free rotation at the loading point, and
(v) inclination in the footing load.

These studies are relegated to future work.

Although the scope of the present study is limited, it is instrumental in providing trends for the contact stress distributions which have hitherto been open to speculation and conjecture. The contact stress distributions are of particular importance to the structural design of footings resting on a granular soil. Within the limits of the experimental study the items of particular interest to structural design applications can be summarized in the following point form.

(a) Where two equally loaded strip footings are to be founded on a layer of granular soil, independent behaviour at working load levels will result when the spacing to width ratio $S/B$ is greater than 4.

(b) In conventional structural design calculations the contact stress distribution beneath a centrally loaded footing is assumed to be uniform. This distribution is assumed to be unaffected by the influence of a neighbouring footing. The results of this series of experiments indicate that when footings approach each other the contact stress distribution is no longer uniform. For structural design calculations, this nonuniform shape can be approximated by an equivalent linear shape where the maximum and minimum contact stresses are given by (Fig. 14)

$$q_1 = q_a \left[ 1 \pm 0.60 - 0.15 \left( \frac{S}{B} \right) \right]$$

where $q_a$ is the average contact stress on each footing.

(c) In conventional structural design calculations, the maximum design flexural moment at the loaded point is given by $q_aB^2/8$. (This is applicable to both isolated and
interfering footings.) If the interference effects are taken into consideration the maximum flexural moment \( M_{\text{max}} \) at the loaded point is given by

\[
M_{\text{max}} = \frac{qL^2}{8} \left[ 1 + \frac{2}{3} \left( \frac{0.60 - 0.15 S}{B} \right) \right]
\]

This represents a percentage increase of 30, 20, and 10% in the maximum bending moment for spacing ratios \((S/B)\) of 1, 2, 3, respectively.

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**References**


