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Influence of residual hydraulic gradients on decay curves for one-dimensional hydraulic pulse tests

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SUMMARY

The hydraulic pulse test is widely used to estimate the hydraulic properties of low-permeability porous media. For the test to be valid, and for Darcy's law to be applicable, the communicating pore space must be saturated. It is also customary to saturate the sample by inducing a steady flow rate through the sample and to neglect any residual hydraulic gradients that may be present in the porous solid due to the saturation process. This paper examines the influence of residual hydraulic gradients on the response of the 1-D hydraulic pulse tests by analytically considering several transient states pertaining to pulse testing in the presence of residual hydraulic gradients that can be present immediately following saturation. Typical numerical results illustrate the effects of residual hydraulic gradients that materialize for certain non-dimensional times governed by a variety of factors, most notably the permeability of the porous medium.

Key words: Geomechanics; Hydrogeophysics; Hydrology; Permeability and porosity.

1 INTRODUCTION

The hydraulic pulse test is recognized as an important development in both laboratory and field testing of the permeability characteristics of low permeability materials. Early studies in this area related to field testing of wells can be found in articles by Muskat (1937), Jacob (1940, 1947), Cooper *et al*. (1967), Bear (1972), Papadopoulos *et al*. (1973), Bredehoeft & Papadopoulos (1980) and others and no attempt will be made to provide a complete review of the topic. References to many theoretical, experimental and field works are given by Barenblatt *et al*. (1990), Selvadurai & Carnaffan (1997), Butler Jr. (1998), Selvadurai *et al*. (2005) and Song & Renner (2006).

1-D pulse tests, which are relevant to the basic theme of this paper, have been conducted under laboratory conditions for the measurement of permeability of rock cores and a key investigation in this area is that of Brace *et al*. (1968), who used the technique to determine the permeability characteristics of Westerly granite under high confining pressures. Companion papers by Hsieh *et al*. (1981) and Neuzil *et al*. (1981) deal, respectively, with the theoretical treatment of 1-D pulse tests and their application to the estimation of permeability of 'tight' rocks from transient test data. Again, in view of space limitations, no attempt will be made to document all the literature on 1-D pulse testing of low permeability materials. A number of important contributions were made by Bernabe (1986, 1987a,b) in connection with the study of effective pressure relationships for low permeability materials such as Chelmsford Granite and Barre Granite. The non-linear volume reduction during isotropic compression enables the estimation of pressure-dependent compressibility of these materials. The experimental results are similar in character to the non-linear pressure permeability relationships obtained recently by Selvadurai & Głowacki (2008) in their studies on Indiana Limestone. Liang *et al*. (2001) have developed approximate solutions for the measurement of permeability from results of pulse tests, accounting for non-linear pressure diffusion. Wang & Hart (1993) have examined the pulse test to estimate the permeability and specific storage of low permeability rocks and indicate a methodology for estimating the relative errors associated with their interpretation from pulse tests. Tokunaga & Kameya (2003) use the transient pressure data to determine specific storage values and the study discusses both theoretical and experimental results. Of interest is the study by Gross & Scherer (2003), who present a dynamic pressurization technique for measuring permeability of cementitious materials.

A key assumption in theoretical developments related to permeability testing in general is that the accessible pore space through which the flow takes place is fully saturated. It is important to note that absence of full saturation of the accessible pore space can lead to erroneous interpretations of permeability derived from both steady state and transient tests. Fig. 1 illustrates results of steady state 1-D permeability tests conducted on Indiana Limestone, where the sample acquires varying degrees of saturation during the application of a steady flow rate. The sample measured 100 mm in diameter and 200 mm in length and the porosity is estimated at 0.166. The steady state pressure established during the application of a

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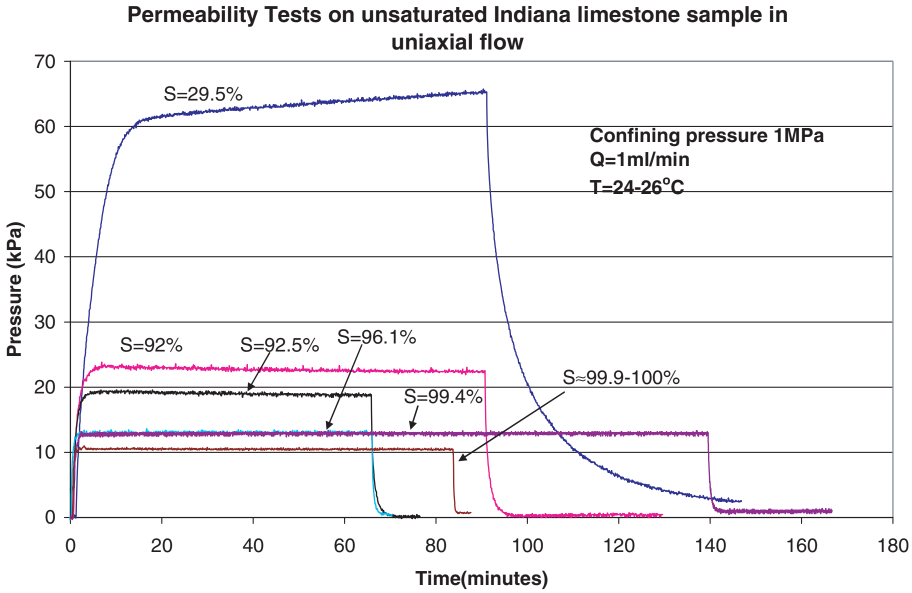

Figure 1. Influence of the degree of saturation on the development of steady state pressure in a 1-D permeability test.

constant flow rate in the sample is directly related to the permeability of the Indiana Limestone, and these results indicate the significant influence of the degree of saturation on the measured value of permeability of rocks. The influence of partial saturation of the pore space of a rock during field permeability measurements is largely an unresolved issue, since desaturation can result if the borehole is exposed to atmospheric pressure. The tacit assumption is that at significant depths the pore space can be fully saturated due to the large hydrostatic pressures that can persist over a geological timescale. In the context of laboratory testing, the approach has been to either subject the test specimen to a hydraulic gradient over a sustained period or to saturate the sample under vacuum. In both approaches, the sample will contain a residual hydraulic gradient immediately after the attainment of either steady flow or full saturation. Therefore, sufficient time must be allowed for the residual hydraulic gradients to dissipate before performing pulse tests on the sample. The disadvantages of such a procedure are twofold; first, the time required for the complete dissipation of the residual hydraulic gradient will depend on the permeability of the porous medium, which, unfortunately, is the property that is being sought, and second, prolonged periods with no fluid flow through the sample and the ends of the sample maintained saturated but open to the atmosphere, can result in the release of air within the pore space, which can negate the primary objective of a saturation sequence. The prudent option is to conduct the hydraulic pulse test immediately following the saturation process, provided account can be taken of the residual hydraulic gradients that can persist after a period of saturation. It should also be noted that, in general, the experimental configurations used for initiating steady flow within a sample are directly adopted to conduct the hydraulic pulse tests: The attainment of zero boundary pressures in a test specimen is not a true indicator that the hydraulic gradients within the test specimen are zero.

The objective of this paper is to examine the role of residual hydraulic gradients on the performance of the 1-D pulse test. This is achieved by examining typical 1-D saturation configurations that can be used to saturate a test specimen, prior to conducting a conventional pulse test.

2 GOVERNING EQUATIONS

Transient phenomena in the hydraulic potential in a fluid saturated porous medium can result either from time-dependent boundary hydraulic potentials or from the diffusive processes associated with the deformability of the porous skeleton and/or the deformability of the pore fluid. In most saturated porous rocks, the fluid flow through the porous medium can be described by Darcy's law and the skeletal deformability characteristics, at stress levels well below the failure stress states, can be described by Hookean elastic behaviour. The general theory of poroelasticity resulting from these idealizations is due to Biot (1941) and, under certain conditions pertaining to the relative compressibility of the pore fluid in relation to the compressibility of the porous skeleton, the hydraulic potential in the pore fluid is governed (see e.g. Bear 1972; Barenblatt *et al*. 1990; Philips 1991; Selvadurai 2000, 2002) by the piezo-conduction or elastic drive equation. In 1-D, the equation can be written as (Hsieh *et al*. 1981)

$$\frac{\partial^2 h}{\partial x^2} = \frac{S_s \mu}{K \gamma_w} \frac{\partial h}{\partial t}, \quad x \in (0, L_0), \tag{1}$$

where $h(x, t)$ is the hydraulic potential (dimensions: L), L_0 is the extent of the domain, μ is the dynamic viscosity (dimensions: $ML^{-1}T^{-1}$; M refers to the dimension of mass and T is the dimension of time), K is the permeability (dimensions: L^2), γ_w is the unit weight of water (dimensions: $ML^{-2}T^{-2}$), S_s is the specific storage of the porous medium (dimensions: L^{-1}), composed of a solid material that is incompressible and defined by

$$S_s = \gamma_w (n C_w + C_{eff}). \tag{2}$$

In (2), n is the porosity and C_w and C_{eff} (dimensions: LT^2M^{-1}) are, respectively, the compressibilities of the pore fluid and the porous skeleton. The partial differential equation (1) for the hydraulic potential is identical to the classical heat conduction equation and can be investigated for several situations of interest to the transient analysis of the 1-D test. It must also be emphasized that the result (1) pre-supposes that the porous medium is fully saturated prior to initiating the transient state; as such, the result cannot be used to investigate transient responses to saturation in an initially dry sample. The governing equation (1) has to be solved subject to certain boundary conditions and an initial condition. At this point, it is important to make the following observations: in order for the diffusive process to achieve a steady state in the flow domain of interest, the domain must necessarily be of finite extent, since the solution for the steady state equivalent of (1) described by the 1-D Laplace's equation exists only for a finite domain. On the other hand, when describing the diffusive processes associated with (1), the analysis of the resulting problem for short relative times is considerably simplified if the porous domain is assumed to be semi-infinite in extent. The concept of a short time cannot be ascribed *a priori* since this is dependent on the permeability of the porous medium, which is as yet an undetermined parameter. With these considerations in mind, we shall make appropriate assumptions concerning the extent of the 1-D domain depending on the nature of the steady state condition and the transient processes involved. These boundary conditions will vary, as does the initial condition required for the solution of (1). The conventional assumption (see e.g. Hsieh *et al*. 1981) is that the hydraulic potential satisfies the initial condition

$$h(x, 0) = 0; \quad x \in (0, L_0). \tag{3}$$

The role of this assumption will be examined in detail particularly with regards to experimental configurations that can be used to estimate the permeability characteristics of the porous medium. At the outset, it must be mentioned that the solution to the elastic drive equation depends, in addition to the permeability of the medium, on the compressibility of the pore fluid, the compressibility of the porous skeleton and the porosity: these parameters are usually determined from separate tests.

3 ANALYSIS OF EXPERIMENTAL CONFIGURATIONS

In this section we shall present the application of the elastic drive equation to examine the performance of 1-D transient tests that are used to estimate the permeability characteristics of low permeability rocks.

3.1 The repressurization and de-pressurization of the boundary of a porous column

We consider the problem of a 1-D domain of a porous region $x \in (0, L_0)$ that has been 'previously saturated' and the hydraulic potential in the column is then allowed to dissipate to zero by maintaining the sample in a saturated condition with the column ends maintained at zero hydraulic potential. The boundary $x = 0$ is subjected to a flow rate Q_0 (dimensions: L^3T^{-1}). The initial boundary value problem is examined by assuming that the finite domain $x \in (0, L_0)$ can be idealized as a semi-infinite domain $x \in (0, \infty)$, for relatively short duration of time. The objective of the test is to determine the time-dependent evolution of the boundary hydraulic potential $h(0, t)$ or $H(t)$. The initial boundary value problem is described by the partial differential equation

$$\frac{\partial^2 h}{\partial x^2} = \omega^2 \frac{\partial h}{\partial t}; \quad x \in (0, \infty); \quad \omega^2 = \frac{S_s \mu}{K \gamma_w} \tag{4}$$

subject to the boundary condition

$$\left(-\frac{\partial h}{\partial x}\right)_{x=0} = \frac{Q_0 \mu}{A K \gamma_w} \mathbf{H}(t), \tag{5}$$

where A is the cross-sectional area of the sample, $\mathbf{H}(t)$ is the Heaviside step function of time, and the initial condition

$$h(x, 0) = 0; \quad x \in (0, \infty). \tag{6}$$

It should be noted that in the solution developed for examining the pressure pulse, it is assumed that the region is semi-infinite. This is not a requirement, but enables the development of a convenient solution. Accordingly, $h(L_0, 0) \equiv 0$ for all $t > 0$. 'This condition is invoked in all the transient solutions developed in the paper'. The solution of the initial boundary value problem is relatively straightforward and can be achieved by applying Laplace transform techniques. The technique will be illustrated for a subsequent problem but for the current purposes it is sufficient to give the final expression for the variation of the hydraulic potential over the semi-infinite region, that is,

$$\frac{h(x,t)}{L_0} = \left(\frac{Q_0 \mu}{A K \gamma_w}\right)\left[\frac{2}{\sqrt{\pi}}\sqrt{\frac{t}{\omega^2 L_0^2}}\ \exp\left(-\frac{\omega^2 x^2}{4t}\right) - \frac{x}{L_0}\text{Erfc}\left(\frac{\omega x}{2\sqrt{t}}\right)\right], \tag{7}$$

where $\text{Erfc}(\zeta)$ is the complementary error function defined by (see e.g. Abramowitz & Stegun 1964)

$$\text{Erfc}(\zeta) = 1 - \int_0^{\zeta} \exp(-\xi^2)\, d\xi, \tag{8}$$

and the length parameter L_0 is introduced to present the results in a non-dimensional form, since there is no natural length parameter associated with a semi-infinite domain. We consider the particular result pertaining to the time-dependent evolution of a hydraulic potential at the boundary $x = 0$, given by

$$\frac{H(t)}{L_0} = \sqrt{\frac{4 Q_0^2 \mu t}{\pi A^2 L_0^2 K \gamma_w S_s}}. \tag{9}$$

At this point, it is pertinent to make the following observation. The pulse test is not a 'fundamental' test in which the decay of the pulse is dependent solely on the permeability of the porous medium. A number of other parameters, including the compressibility of the pore fluid (C_w), the compressibility of the porous skeleton (C_{eff}), the compressibility of the solid material (C_s) and the porosity (n) of the medium can all influence the decay rate. For most geologic media, the influence of C_s can be neglected in comparison with the other compressibilities of the system. The value of C_w applicable to the fluid at a given temperature and range of pressure can be obtained from published physical data. The value of C_{eff} can be obtained by conducting either volume compression tests or triaxial or uniaxial tests with axial and radial strain measurement. This still leaves the porosity n as a parameter that needs to be determined. The accurate estimation of porosity is not routine and this requires access to mercury intrusion porosimetry and other techniques. The possibility exists to estimate the void volume in the sample by appeal to mixture theories but such estimates are generally inconclusive, since the total void space does not necessarily reflect the void space accessible to fluid flow. Therefore, for the rigorous application of the pulse test as a means of determining the permeability, the additional parameters should be known *a priori*. This is indeed a limitation and the test should be considered as a confirmatory test for permeabilities determined via other techniques. The only permeability test that does not involve other extraneous parameters is the constant flow rate test; unfortunately, with low-permeability materials, such a test can be time consuming particularly with respect to establishing a reliable steady state. In a 1-D flow experiment, all the parameters in (9) are known except the permeability K, which implies that the hydraulic potential response for a short relative time can be used to estimate K. The constraint on the short relative time can be obtained by examining the leading term in (7) that is evaluated at the boundary $x = 0$, which is given by (see e.g. Hsieh *et al*. 1981)

$$\left(\frac{tK\gamma_{\mathrm{w}}}{S_{\mathrm{s}}\mu L_0^2}\right) < 1. \tag{10}$$

If the flow rate Q_0 is maintained for a long time, defined by the parameter in (10), steady state flow is established and the hydraulic potential distribution in the 1-D sample is given by

$$h(x) = \left(\frac{Q_0 L_0 \mu}{AK\gamma_{\mathrm{w}}}\right)\left(\frac{L_0 - x}{L_0}\right) = h_0\left(\frac{L_0 - x}{L_0}\right). \tag{11}$$

We now consider the situation where the flow rate is terminated when steady state conditions are reached. In this case, the pressure decay at the boundary $x = 0$ of the 1-D porous region is governed by the initial boundary value problem for which the governing partial differential equation is (1); the initial condition is

$$h(x, 0) = H_0\left(\frac{L_0 - x}{L_0}\right), \quad x \in (0, L_0), \tag{12}$$

and the single boundary condition is

$$\Phi\left(\frac{\partial h}{\partial x}\right)_{x=0} = \left(\frac{\partial h}{\partial t}\right)_{x=0}, \quad \Phi = \left(\frac{AK}{\mu V_{\mathrm{w}} C_{\mathrm{w}}}\right), \tag{13}$$

where V_{w} is the volume of fluid that is contained in the region used to supply the fluid flow to one end of the sample. The solution of the initial boundary value problem defined by (1), (12) and (13) can be obtained by employing a Laplace transform technique. Applying the Laplace transform to (1) and making use of the initial condition (12), we obtain the general result

$$\bar{h}(x, s) = A(s)\exp(-\omega x\sqrt{s}) + B(s)\exp(\omega x\sqrt{s}) + H_0\left(\frac{L_0 - x}{sL_0}\right), \tag{14}$$

where $\bar{h}(x, s)$ is the Laplace transform of $h(x, t)$defined by

$$\bar{h}(x, s) = \{h(x, t); t\} = \int_0^\infty h(x, t)\exp(-st)\,\mathrm{d}t, \tag{15}$$

and $A(s)$ and $B(s)$ are arbitrary functions of the transform parameter s. Since the porous domain occupies the region $x \in (0, \infty)$ we require, for regularity conditions, $B(s) \equiv 0$. Employing the boundary condition (13), we can determine the unknown function $A(s)$ and, by substituting this in (14), we obtain an explicit expression for $\bar{h}(x, s)$. Since we are primarily interested in the time dependent variation of the hydraulic potential at the boundary $x = 0$ of the porous domain, we obtain

$$\frac{\bar{h}(0, s)}{H_0} = \frac{1}{s} - \frac{\Phi}{L_0}\left(\frac{1}{s^{3/2}(\Omega + \sqrt{s})}\right), \tag{16}$$

$$\Omega = \Phi\omega. \tag{17}$$

Inverting (16) and denoting $h(0, t)$ by $H(t)$ we have

$$\frac{H(t)}{H_0} = 1 - \frac{1}{\Omega\omega L_0}\left\{2\sqrt{\frac{\Omega^2 t}{\pi}} - \left[1 - \exp(\Omega^2 t)\,\mathrm{Erfc}(\sqrt{\Omega^2 t})\right]\right\}. \tag{18}$$

It must be noted that the derivation of this explicit expression for the decay of the hydraulic potential at the pressurized boundary assumes that the porous domain is semi-infinite. As such, the application of the limit $t \to \infty$ is inadmissible. If all the relevant parameters except the permeability of the porous medium are known, then the decay pattern can be used to estimate the permeability of the porous medium. Alternatively, the results derived from the previous stages of the experiment could be used to predict the transient response associated with the pressure decay.

3.2 1-D pulse test–pulse flow in the direction of the steady flow

In this test, we consider the situation where the 1-D element is first subjected to steady flow to initiate saturation. This can be done by pressurization or vacuum saturation or by maintaining a hydraulic gradient similar to (12). In the pressurization mode, it is imperative that the sample is completely saturated to ensure that Darcy's law is applicable. We shall first consider the case where the sample is subjected to a constant hydraulic potential h_0 over the entire length (Fig. 2). The pulse test is conducted by applying a hydraulic pulse in the form of a Dirac delta function of time such that

$$h(0, t) = h_0 + H_0\,\delta(t), \tag{19}$$

where $\delta(t)$ is the Dirac delta function. The resulting initial boundary value problem applicable to the semi-infinite domain $x \in (0, \infty)$ is governed by the partial differential equation (11), the initial conditions

$$h(x, 0) = h_0, \quad x \in (0, L_0), \tag{20}$$

and the boundary condition and the added definition

$$\Phi\left(\frac{\partial h}{\partial x}\right)_{x=0} = \left(\frac{\partial h}{\partial t}\right)_{x=0}, \tag{21}$$

$$h(0, 0) = h_0 + H_0\,. \tag{22}$$

We note here that h_0 is the maximum residual hydraulic potential at the point of application of the hydraulic pulse. The solution of the resulting initial boundary value problem applicable to the semi-infinite domain is elementary and can be easily achieved using the Laplace transform technique outlined previously. The time-dependent decay of the hydraulic head at the boundary of the semi-infinite domain is given by

$$\frac{\tilde{H}(t)}{H_0} = \frac{H(t) - \{h(0, 0) - H_0\}}{H_0} = \exp(\Omega^2 t)\,\mathrm{Erfc}(\sqrt{\Omega^2 t}). \tag{23}$$

Therefore, the initial constant hydraulic potential is simply added to the hydraulic potential created by the application of a boundary potential in the form of a Dirac delta function. For this reason, the analysis of the pulse test neglects any initial constant hydraulic potential that may be present in the porous medium.

We now modify the initial boundary value problem to consider the case where the 1-D region is first saturated by inducing flow initiated by a gradient similar to that defined by (12) (Fig. 3), that is,

$$h(x, 0) = h_0\left(\frac{L_0 - x}{L_0}\right), \quad x \in (0, L_0). \tag{24}$$

The hydraulic pulse test is conducted by applying a pulse in the form of a Dirac delta function of time such that the boundary

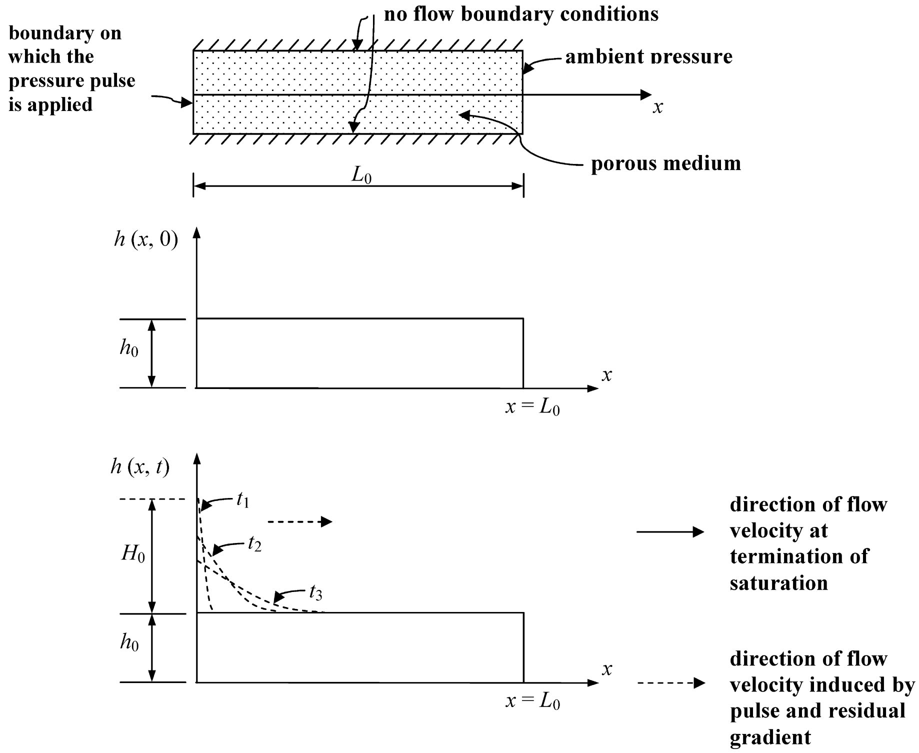

Figure 2. Hydraulic pulse applied to a 1-D element with constant initial hydraulic potential.

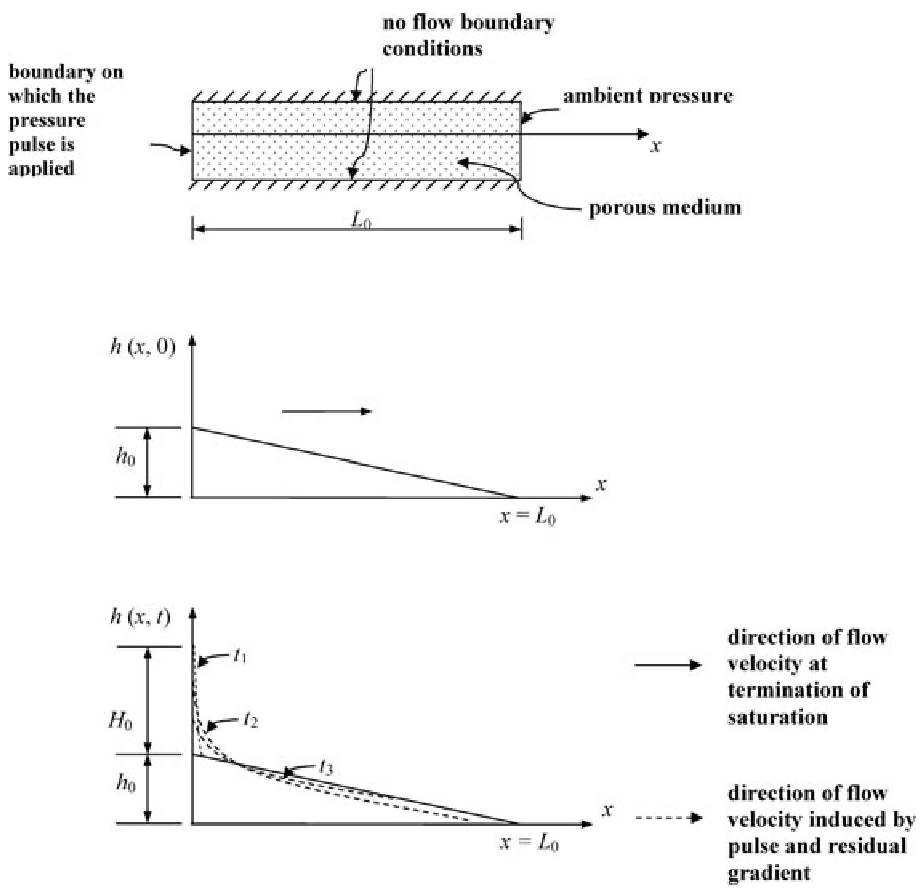

Figure 3. Hydraulic pulse applied to a 1-D element with a linear initial hydraulic potential: pulse associated flow in the direction of the initial flow.

condition is identical to (19). The resulting initial boundary value problem for the dissipation of the hydraulic potential in the system is governed by the partial differential equation (1), the initial condition (23), the boundary condition (20) and the added definition (22). The resulting solution to the time-dependent decay of the hydraulic potential at the location where the hydraulic potential pulse is applied is given by the superposition of the solutions (18) and (23) (with h_0 set to zero in the latter): using the definition of $\tilde{H}(t)$ given in (23), we obtain

$$\frac{\tilde{H}(t)}{H_0} = \exp(\Omega^2 t)\,\mathrm{Erfc}\big(\sqrt{\Omega^2 t}\big) - \frac{h_0}{H_0}\left[\frac{1}{\Omega\omega L_0}\left\{2\sqrt{\frac{\Omega^2 t}{\pi}} - \left[1 - \exp(\Omega^2 t)\,\mathrm{Erfc}(\sqrt{\Omega^2 t})\right]\right\}\right]. \tag{25}$$

The superposition of solutions is valid since the kinematic constraints related to the expansion of the fluid contained in the pressurized region as the fluid migrates to the porous region is the same in both situations. The important observation is that, unlike the case where the initial hydraulic potential is a constant, in this case the time-dependent decay of the hydraulic potential at the boundary has an added influence due to the initial gradient (24). Again the resulting expression for the time-dependent decay of the hydraulic potential at the boundary is influenced by the initial non-uniform distribution of hydraulic potential over the length of the 1-D element.

3.3 1-D pulse test–pulse flow opposite to the direction of the steady flow

We consider the case where the 1-D porous medium is subjected to steady flow by the application of a hydraulic potential h_0 at the location $x = L_0$ and the location $x = 0$ is maintained at zero potential (Fig. 4). This state serves as the initial condition

$$h(x, 0) = h_0\left(\frac{x}{L_0}\right), \quad x \in (0, L_0). \tag{26}$$

The hydraulic pulse is applied at the location $x = 0$ such that the boundary pulse is defined by

$$h(0, t) = H_0\,\delta(t). \tag{27}$$

The resulting initial boundary value problem governing the diffusion of the hydraulic pulse within the semi-infinite domain is defined by the partial differential equation (1), the initial condition (26), the boundary condition (21) and the boundary condition (27). The solution to this initial boundary value problem can be obtained quite conveniently using a Laplace transform technique, and takes the form

$$\frac{\tilde{H}(t)}{H_0} = \exp(\Omega^2 t)\,\mathrm{Erfc}\big(\sqrt{\Omega^2 t}\big) + \frac{h_0}{H_0}\left[\frac{1}{\Omega\omega L_0}\left\{2\sqrt{\frac{\Omega^2 t}{\pi}} - \left[1 - \exp(\Omega^2 t)\,\mathrm{Erfc}\big(\sqrt{\Omega^2 t}\big)\right]\right\}\right]. \tag{28}$$

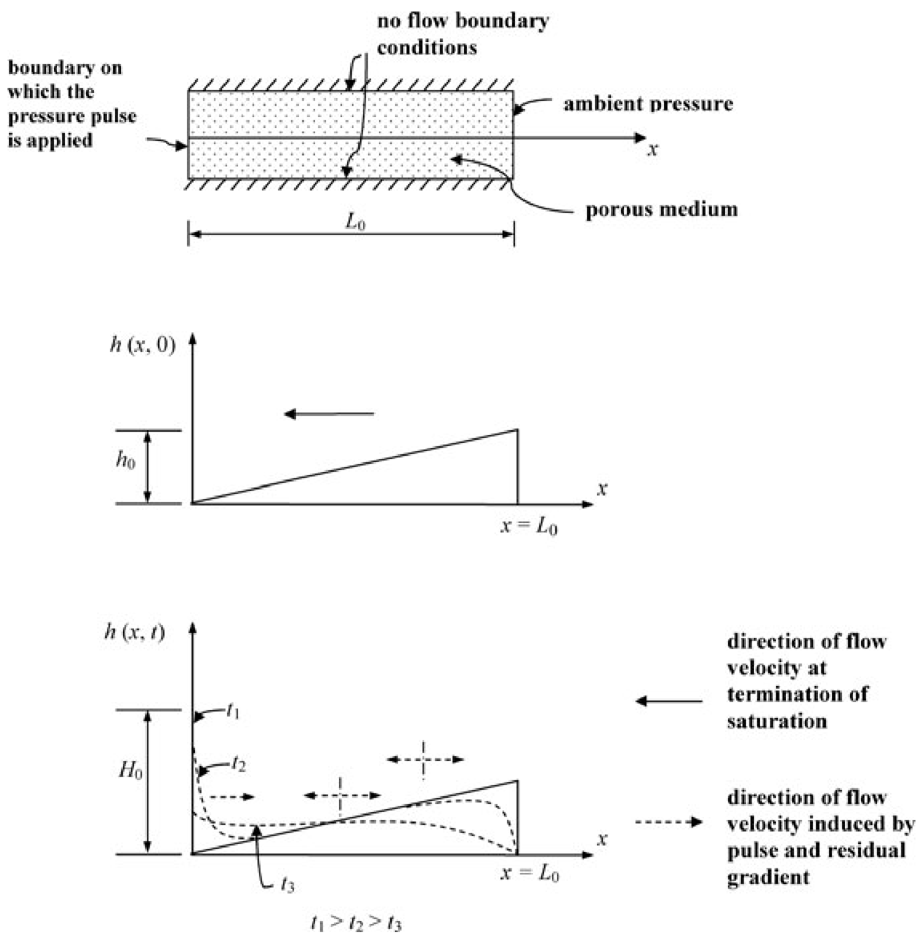

Figure 4. Hydraulic pulse applied to a 1-D element with a linear initial hydraulic potential: pulse associated flow opposite to the direction of the initial flow.

Again we observe that the non-uniform residual hydraulic potential present at the end of a saturation sequence has an influence on the decay of the hydraulic potential that is applied to the 1-D element.

3.4 1-D pulse test–residual hydraulic head of a parabolic form

We consider the situation where the 1-D porous region is saturated with a constant potential and this potential is reduced to zero at the two boundaries. The excess hydraulic potential will dissipate according to the classical solution for the diffusion equation which can be represented in the form of an infinite series (Selvadurai 2000). To illustrate the influence of a non-uniform initial hydraulic head on the results of a pulse test, we assume that the initial hydraulic head can be represented in the parabolic form

$$h(x,0) = h_0 \frac{x(L_0 - x)}{L_0^2}, \tag{29}$$

with the peak residual hydraulic head occurring at the mid section of the 1-D element. The hydraulic pulse is now applied at the boundary $x = 0$. We focus on the evaluation of the time-dependent dissipation of the hydraulic head at the point of application of the pulse. The initial boundary value problem governing this problem is defined by the partial differential equation (1) subject to the kinematic boundary condition (20), the initial condition (29) and the condition for the definition of the hydraulic pulse given by (27). The solution of the resulting initial boundary value problem can be examined using a Laplace transform technique and the details will not be pursued here; the final result for the time-dependent decay of the hydraulic potential at the point of application of the hydraulic pulse is given by

$$\begin{aligned}\frac{H(t)}{H_0} &= \exp(\Omega^2 t)\,\mathrm{Erfc}\big(\sqrt{\Omega^2 t}\big)\\ &\quad + \frac{2h_0}{H_0}\left[-\frac{2t}{\omega^2 L_0^2} + \left(\frac{\Phi}{\Omega^2 L_0} + \frac{1}{(\Omega\omega\, L_0)^2}\right)\left\{2\sqrt{\frac{\Omega^2 t}{\pi}}\right.\right.\\ &\quad \left.\left. - \left[1 - \exp(\Omega^2 t)\,\mathrm{Erfc}\big(\sqrt{\Omega^2 t}\big)\right]\right\}\right] \end{aligned} \tag{30}$$

The analysis can be extended to cover other forms of residual hydraulic potential distribution that could be present if insufficient time is allowed for the dissipation of the hydraulic potentials required to saturate the 1-D element. This will not add to the basic objective of the paper, which is to demonstrate the influences of the residual hydraulic gradients on the time-dependent decay of a hydraulic pulse applied to a 1-D element of finite extent. It is noted that, in the event that the porous medium has no residual hydraulic potentials, the results (22), (25), (28) and (30) all reduce to the classical result pertaining to the decay of a pressure pulse applied at the boundary of a saturated porous medium of semi-infinite extent.

4 NUMERICAL RESULTS

In this section, we present certain typical numerical results, purely with a view to establishing the influence of the residual hydraulic potential on the decay of the potential pulse applied at the boundary of the 1-D region. There are a number of parameters that could be considered, such as C_w, μ and γ_w, which are properties of water at a particular temperature; A and L_0, which are specimen size dependent; K and S_s, which are porous medium dependent; V_w, which is test arrangement dependent and h_0/H_0, which is test dependent. However, consideration of the variation of all these parameters for numerical illustration is perhaps unwarranted, in view of the exact closed form nature of the final expressions for the hydraulic potential decay. It is convenient to select the non-dimensional parameters $T(= \Omega^2 t)$, $\omega L_0 \Omega$ and h_0/H_0 as the basic parameters that can be varied to assess the influence of the residual hydraulic gradients. To provide a basis for comparison, we can interpret the numerical results in relation to a 1-D pulse test conducted on a cylindrical sample of diameter 100 mm and length 150 mm, with a fluid volume of 10 ml in the pressure measuring system, along with the following mechanical and physical parameters:

$$A \approx 7.5 \times 10^{-3}\ \mathrm{m}^2;\quad V_w \approx 10^{-5}\ \mathrm{m}^3;\quad n \approx 0.2,$$

$$C_w \approx 4.5 \times 10^{-7}\ \mathrm{m}^2\ (\mathrm{kN})^{-1};\quad C_{\mathrm{eff}} \approx 1.0 \times 10^{-7}\ \mathrm{m}^2\ (\mathrm{kN})^{-1};$$

$$\mu \approx 10^{-6}\ \mathrm{kNsm}^{-2}.$$

For this experimental situation,

$$T = \Omega^2 t = \left(\frac{A^2 K (nC_w + C_{\mathrm{eff}})\, t}{\mu\, V_w^2\, C_w^2}\right) \approx 5 \times 10^{17}\ K\, t, \tag{31}$$

where the permeability K is expressed in m^2 and the time t is expressed in s. In a typical estimate for a material with permeability $K \approx 10^{-18}\ \mathrm{m}^2$, a non-dimensional time factor of $T \approx 5$ can be attained at a time $t \approx 10$ s; similarly, for a material with permeability $K \approx 10^{-20}\ \mathrm{m}^2$, a non-dimensional time factor of $T \approx 5$ can be attained at a time $t \approx 1000$ s. The second non-dimensional parameter $h_0/H_0 \omega L_0 \Omega$ accounts for characteristics of the experimental configuration and the magnitude of the hydraulic pulse (H_0) in relation to the peak hydraulic potential (h_0) associated with the residual hydraulic gradient. Again, for this hypothetical experimental configuration

$$\left(\frac{h_0}{H_0 L_0 \Phi \omega^2}\right) \approx \frac{1}{50}\left(\frac{h_0}{H_0}\right). \tag{32}$$

For purposes of illustration, we select values of $(h_0/H_0 L_0 \Phi \omega^2) =$ 1/5 and 1/10. Fig. 5 illustrates analytical results for the decay of the hydraulic potential in a 1-D pulse test where the hydraulic gradient induced by the pulse has the same sign as the initial hydraulic gradient used to saturate the sample. These results indicate that the initial hydraulic gradient has an appreciable influence on the decay pattern observed in a typical test. The decay characteristics can be completely altered if the parameter $(h_0/H_0 L_0 \Phi \omega^2)$ becomes large. This is indicative of hydraulic pulse tests conducted with pressures significantly lower than the maximum hydraulic potentials that are applied to saturate the sample. Figs 6 and 7 illustrate corresponding analytical results on the decay of the hydraulic potential in a 1-D pulse test where the hydraulic gradient induced by the hydraulic pulse has a sign opposite to the hydraulic gradient applied to saturate the sample. Here again, the residual hydraulic gradient has a significant effect on the decay pattern and the relative magnitudes of the peak hydraulic potential in relation to the peak value applied to conduct the hydraulic pulse test.

5 CONCLUDING REMARKS

The pulse test has great utility in conducting relatively rapid tests to investigate the permeability characteristics of low-permeability materials. The implicit assumption in almost all theoretical investigations of the pulse test is that the material being tested is void of

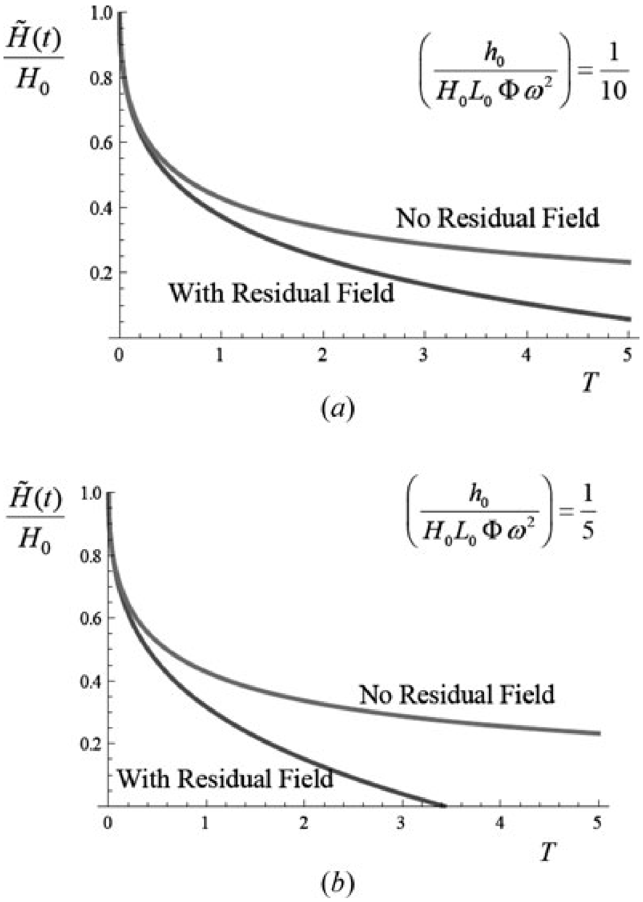

Figure 5. Influence of initial hydraulic gradients on the results of conventional 1-D pulse tests. [Initial hydraulic gradient has the same sign as the gradient induced by the hydraulic pulse. Result corresponding to *equation* (25).] (*a*) $(h_0/H_0L_0\Phi\omega^2) = 1/10$; (*b*) $(h_0/H_0L_0\Phi\omega^2) = 1/5$, and this 1-D parameter is define by (32).

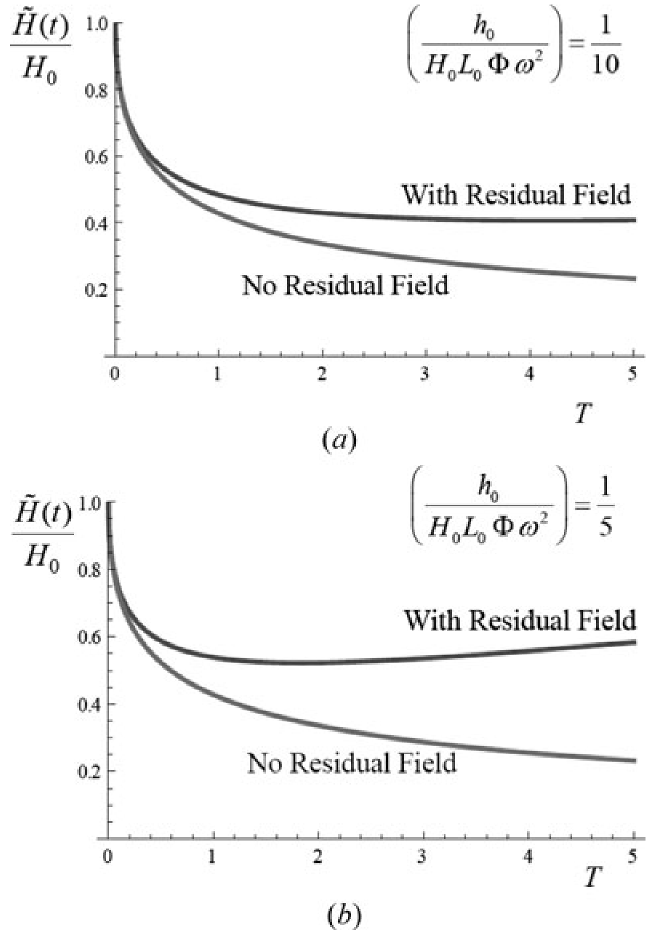

Figure 6. Influence of initial hydraulic gradients on the results of conventional 1-D pulse tests. [Initial hydraulic gradient has a sign opposite to the gradient induced by the hydraulic pulse. Result corresponding to *equation* (28).] (*a*) $(h_0/H_0L_0\Phi\omega^2) = 1/10$; (*b*) $(h_0/H_0L_0\Phi\omega^2) = 1/5$, and this 1-D parameter is define by (32).

any hydraulic potentials that have been applied to fully saturate the sample. The degree of saturation can have a significant influence on both the steady state and transient responses of the permeability tests. The assumption of zero residuals of initial hydraulic gradients is a valid one provided sufficient time is allowed for the dissipation of the initial hydraulic transients or, for that matter, hydraulic transients induced by the repetition of pulse tests. In order to correctly estimate the time required for the dissipation of the hydraulic transients, a prior knowledge of the permeability of the material is essential; this is unfortunately the property under investigation. Furthermore, prolonged time to allow for dissipation of initial hydraulic transients can have the effect of de-saturating the sample by the release of air, which negates the saturation process. This study proposes the incorporation of initial hydraulic transients in the assessment of the conventional pulse test results. The paper presents exact closed form solutions that can be used to examine the influence of initial hydraulic gradients that can be present after the saturation phase of a 1-D sample. The presence of a constant initial hydraulic potential has no influence on the pulse test, whereas an initial hydraulic gradients does. The results presented in this paper can be used either to examine the pulse test in the light of any initial hydraulic gradients that may be present or to decide on the magnitude of the peak hydraulic potentials used in the saturation process

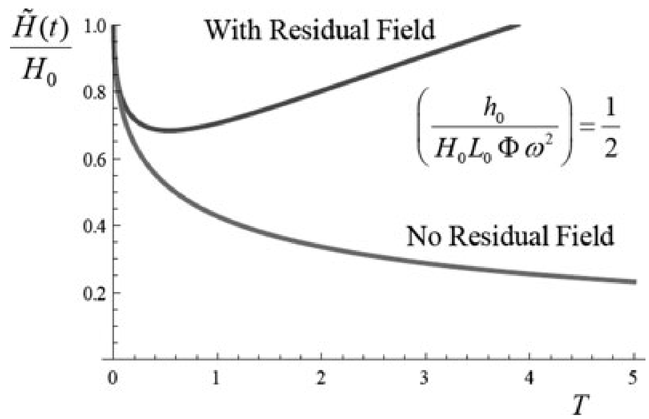

Figure 7. Influence of initial hydraulic gradients on the results of conventional 1-D pulse tests. [Initial hydraulic gradient has a sign opposite to the gradient induced by the hydraulic pulse. Result corresponding to *equation* (28) with $(h_0/H_0L_0\Phi\omega^2) = 1/2$, and this non-dimensional parameter, is define by (32).]

in relation to the peak hydraulic potential applied to conduct a pulse test so that the influence of the residual hydraulic transients can be neglected. The current practice of allowing the boundary pressures of the 1-D test specimen to reduce to zero before conducting a pulse test is counter productive to the objectives of a saturation phase and furthermore the attainment of zero boundary potentials does not imply that the sample is void of initial hydraulic transients. The influence of heterogeneous hydraulic properties can of course compound the problem of ensuring that the pulse tests are conducted in a medium that is free of initial hydraulic transients. Also, the question of extending this type of analysis to the consideration of initial hydraulic gradients encountered in radial flow experiments has been raised by a reviewer. The extension is certainly possible provided the planar fluid flow region being investigated is bounded (e.g. annular). This is a mathematical constraint necessary and sufficient to enable the development of a steady state in the region. As such, the analysis would not apply to the planar fluid flow problem associated with a borehole in a medium of infinite extent. The techniques can be applied for pressurization of a borehole over a finite length since the problem is now axisymmetric and 3-D. A convenient analytical solution can also be obtained for pulse tests conducted in a spherical cavity in a porous medium of infinite extent.

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