Can Financial Intermediation Induce Endogenous Fluctuations?*

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Abstract

This paper studies the possibility of endogenous fluctuations caused by activities of financial intermediaries. Risk-averse agents borrow from banks and invest in a risky two-state capital technology. The probability of success with the technology is assumed to be decreasing in the amount of capital invested. In a complete information setting with intermediation, the efficient loan contract achieves complete risk sharing but the amount invested in the risky project is smaller than the loan size. This "income effect" is responsible for the endogenous generation of complex dynamics. In the absence of intermediation, the economy studied cannot exhibit any cyclical fluctuations.

Key words: Financial intermediation, endogenous fluctuations, loan contracts.

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1 Introduction

It is now virtually undisputed that banks and financial intermediaries play a crucial role in the savings-investment process. A relatively large literature has also emerged that successfully connects banking activity with the propagation and amplification of output fluctuations in modern economies. Within the context of this large canvas, this paper asks a simple question: can banks *introduce* output fluctuations in an economy where there were none in their absence? While we applaud banks for promoting growth in output, are they to be blamed for introducing output volatility in an otherwise calm economy?

More specifically, this paper examines the possibility that in a complete information world, an economy's financial setup may itself be the *raison d'etre* for endogenous cyclical fluctuations. It establishes that efficient risk-sharing by financial intermediaries may expose an economy to a wide variety of endogenous fluctuations. On the contrary, in the complete absence of intermediaries, the economy studied exhibits a smooth monotonic transition to a stationary state of real activity, and is hence immune to cyclical fluctuations.

The economy we study is similar in some respects to the one explored in Azariadis (1993). In particular, the setting is a production economy of the Diamond (1965) variety inhabited by overlapping generations of risk-averse agents. There is a single final good produced according to a standard neoclassical production function using capital and labor as inputs. In contrast to the specifications of the standard Diamond model however, there is one major modification. Here, the output technology is a two-state (success and failure) stochastic process and the probability distribution of uncertain future output depends on the amount of capital invested by the agent. Capital has a dual role: on the one hand, more capital investment *decreases* the probability of success with the output technology (henceforth "project"), while on the other hand, through standard channels, more capital increases output in the success state.

In such an environment where risk-averse agents face idiosyncratic income shocks, it is natural to think that financial intermediaries would arise to intermediate savings, and assured by the law of large numbers, achieve complete risk sharing.[1] However, the fact that project success or failure is dependent on the amount of capital invested, and the fact that limited liability protects borrowers in case of default, jointly necessitate some kind of control the bank has to exert on the capital investment process. Following Holmstrom and Tirole (1997), we assume the existence of an *active* intermediation regime, one characterized by intermediaries that lend only to borrowers who accept restrictions on their own actions as stipulated by the intermediary within the context of a contractual arrangement. Agents seek to borrow funds in order to make capital investments on their projects. At the same time, they make deposits with the intermediary which promises them a safe default-free rate of return. We prove that efficient risk sharing will require that an agent invests *less* than the entire amount of the loan he receives.[2] In other words, the agent is provided with sufficient funds not only to carry out the entrepreneurial activity, but also to hedge his bet. This may be explained as follows. On the one hand, provision of complete insurance requires disbursement of a loan of sufficient size such that the recipient can achieve full consumption smoothing across different states of nature. On the other hand, because of limited liability, a big loan may prompt "overinvestment" in the capital technology thereby reducing the chance of success. One way the efficient contract trades off these two conflicting concerns is by making the loan size high and stipulating a low capital investment. Thus the agent is provided additional resources for use in the first period of his life which is the source of what we label the "income effect". An interesting property of our model is that the functional relationship between the size of the loan and the size of the

[1] Intermediaries have been argued to perform several other important functions, delegated monitoring, screening, etc., to name a few. A fuller description of these functions is available in Pagano (1993). In this paper, we focus exclusively on the risk-sharing function of financial intermediaries.

[2] It is important to emphasize that we abstract away from issues of moral hazard and adverse selection here. In other words, capital investments by agents are fully observable and the probability distribution of uncertain future output is the same across all agents. In particular, we assume that financial intermediaries can perfectly observe, costlessly monitor and provide verifiable evidence (to courts of law) of the capital investments of borrowers.

capital investment (as part of the efficient contract) can be locally non-monotone. Indeed, the strength of this income effect can change over time and this has important consequences.

We go on to establish that under some conditions on the probability distribution and its elasticity with respect to the amount of capital invested, the aforementioned "income effect" is strong, and this causes the law of motion for the aggregate capital stock to be non-monotonic. By means of numerical examples, we are also able to demonstrate the existence of a large class of endogenous output fluctuations of varying periodicity. Indeed, the possibility of development traps and aperiodic (chaotic) dynamical equilibria also arises. Furthermore, to bring out the contrast sharply, we prove that our economy in the absence of any form of intermediation, much like a non-pathological Diamond model, cannot produce any fluctuations in output.

A few words on the underlying nature of these fluctuations are in order. Focus on a starting situation where the capital stock in the economy is relatively high. Then, the wage rate in the economy is high, and so are saving and desired investment as fractions of wage income. Left unfettered, if actual investment in the risky capital technology were high, the riskiness of that technology would be very high, and overall expected return would be very low. The bank, recognizing the problem, steps in and disburses a large loan and dictates a low *actual* investment so as to keep the riskiness low, and expected return high. So, the amount invested in the capital technology is low and the amount invested in the safe deposits with the bank is high. The output in the following period is then low (given the low actual investment). Then next period's wage rate is low, and so are saving and desired investment as fractions of wage income. In the absence of the bank, a low wage income would imply a low desired investment and hence low riskiness but low expected return. Under our assumptions on the success probability function, the bank may be able to avoid this bad situation by disbursing a small loan and stipulating a high actual investment. The high investment may raise the expected return to investing in the capital technology, and raise output next period, and so on.

It is also important to emphasize that these cycles emerge in a relatively standard economy; in particular, we do not take recourse to elements like limited market participation, imperfect competition, multiple sectors etc. which have been shown to contribute to cyclical fluctuations (see Boldrin and Woodford (1990) for a survey). We do however emphasize the role of the "income effect" in the generation of cyclical fluctuations, although it should be noted that the *nature* of the income effect we describe is very different from the one explored in Azariadis (1981) or Grandmont (1985). These authors stipulate exogenous restrictions on preferences that generate a sufficiently strong income effect which, in turn, produces "backward-bending" savings/labor-supply functions. In our setup, the income effect is endogenously generated as a by-product of efficient risk sharing in the presence of limited liability.

The possibility that financial intermediation, in and of itself, may expose an economy to endogenous fluctuations has long been recognized.[3] Such a link was at the heart of John Mills' 1867 celebrated paper entitled "On Credit Cycles and the Origin of Commercial Panics" as well as at the center of a prolonged debate between the Currency School and the Banking School in England in the early-to-mid 19th century.[4] Taking his cue from these debates, Friedman (1960) in fact advocated 100% reserve requirements on financial intermediaries as a sure step towards eliminating "excessive economic fluctuations". Friedman's concern was criticized by Sargent and Wallace (1982) who constructed a model in which unfettered intermediation leads to endogenous fluctuations that are Pareto optimal. Smith (1991) reconsidered Friedman's proposal in a world with financial intermediaries and nominal assets. Our work differs from these abovementioned investigations in that our focus is entirely on real fluctuations as opposed to volatility in the price level.

[3] Friedman and Schwartz (1963) provide compelling evidence that most of the pre-World War II recessions were associated with major transfers of resources out of the banking system ("disintermediation") and into various other assets, thereby suggesting that the financial system may itself have been the source of the economic volatility.

[4] See Smith (1988) for an insightful and modern discussion.

It is important to note that our question and setup are quite different from much of the existing literature that connects financial development and output variability. First, *unlike* Bernanke and Gertler (1989), Bachetta and Caminal (2000), among others, we are interested in demonstrating how banking activities may *create* the possibility of cyclical fluctuations, rather than how they propagate or amplify *existing* output disturbances. Second, our focus is different from a large class of models that rely on the "cycle of overlending" to explain the recent financial crises crippling Asia and Latin America.[5] Third, unlike all of this literature, we deal with a complete information world in which risk-averse agents confront a risk-neutral lender; in particular, there is no moral hazard ex-ante or ex-post, unlike Williamson (1987), or Aghion, Banerjee, and Piketty (1999), or adverse selection as in Azariadis and Smith (1996). Den Haan, Ramey, and Watson (2003) consider long-term lending relationships that are subject to moral hazard and other non-informational frictions. They establish a complementarity between financial intermediation and investment (somewhat similar in spirit to ours) which cause multiple Pareto-ranked equilibria to emerge in their framework. However, their focus is on magnification and propagation under moral hazard which we abstract away from.

The plan for the rest of the paper is as follows. In the next section, we describe the model's environment, i.e., agents' preferences and technology, and then we set up the goods and labor markets. In Section 3, we derive the efficient financial contract between a risk-averse borrower and a bank that can control the borrower's actions. In Section 4, we define a general competitive equilibrium in our economy and go on to study stationary and non stationary equilibria. Here, we demonstrate by means of numerical examples that our economy exhibits complex dynamics, and also prove that our economy absent banks, is incapable of generating any endogenous output variability. Section 5 concludes. Proofs of certain important results are contained in the appendices.

[5] For example, according to Kaminsky and Reinhart (1996), "a financial crisis occurs as an economy enters a recession that follows a prolonged boom in economic activity fueled by credit creation and surges in capital inflows."

2 The Model

2.1 Environment

We study a simple production economy that is inhabited by an infinite sequence of two-period lived overlapping generations, plus an initial old generation. Time is discrete and is indexed by $t = 1, 2, ...$ In each period, a new generation comprising a continuum of agents with measure one is born. Each generation is identical in size and composition.

Only the initial old is endowed with a capital stock of $K_1 > 0$. All other agents are endowed with one unit of time only when young. Agents do not value leisure; as a result, every agent supplies one unit of labor inelastically to the labor market when young, at the going wage rate, w_t. There is a single perishable good in the economy.

The preferences of all agents are summarizable by an atemporal additively-separable utility function, U, where

$$U(c_1, c_2) = u(c_1) + v(c_2) \tag{1}$$

and c_j, $j = 1, 2$ refers to consumption of the good in the jth period of life. The functions u and v are strictly increasing, strictly concave, and smooth. All individuals are risk-averse. Additionally,

Assumption 1

$$-\frac{c_2 v''(c_2)}{v'(c_2)} \leq 1 \tag{2}$$

holds, i.e., the Arrow-Pratt measure of relative risk-aversion is less than or equal to unity. This assumption ensures that optimal saving is non-decreasing in its rate of return. Its importance will be evident in Section 4 below.

Agents can transfer income across time periods by investing in one of two possible assets. The first is a riskless asset, where a unit of the good invested at t returns a gross amount r_{t+1} at the start of $t+1$ for sure. The exact nature of this asset will be described below. The second avenue for the intertemporal transfer of income is investment in a commonly accessible risky output technology. This technology is summarized by a constant returns to scale neoclassical production function $F(.)$ that employs labor (N) and capital (K) as inputs. We let $x_t \equiv \frac{K_t}{N_t}$ denote the capital to labor ratio, and $f(x_t) \equiv F(\frac{K_t}{N_t}, 1)$, denote the associated intensive production function. The function $f(.)$ is assumed to satisfy $f(0) = 0$, $f' > 0 > f''$, and standard Inada conditions. Capital depreciates 100% between periods.

We modify the standard Diamond (1965) formulation by postulating that a capitalist must invest before he hires labor. If the investment turns out to be "successful", then labor is hired, and subsequent production of the final good takes place. If an investment "fails", then the net output is zero, and no labor is hired. The amount that an individual invests in period t is denoted by K_{t+1}. The outcome of the investment is subject to a idiosyncratic random event. The probability that an individual is successful is assumed to be dependent on the amount he invests, and is denoted by $p(K_{t+1})$. The probability that his project is a failure is therefore given by $1- \ p(K_{t+1})$. At the start of $t+1$, the idiosyncratic uncertainty is resolved, and each old capitalist then knows whether his investment project is a success or a failure. By "failure", we mean that the amount invested, K_{t+1}, becomes useless to him, and to all others, and he will not hire any worker. Given the above characterization, it may be convenient to think of the project as an R&D activity. For any given individual i, the expected output from investing K_i is $p(K_i)F(K_i, \cdot)$. For the economy as a whole, however, there is effectively aggregate certainty, and the aggregate effective capital stock is $p(K)K$, giving rise to the certain output $F(p(K)K, N)$.

We make the following assumptions about the function p.

Assumption 2 *a)* $1 \geq p(0) > 0$, *and* $p(K) \in (0, 1) \ \forall K > 0$

b)

$$p'(K) < 0 \quad \forall K > 0 \tag{A.1}$$

and $p'(0) < \infty$.

This assumption on p' is somewhat analogous to those made by de Meza and Webb (1999, 155).[6] In plain English, the chance of forfeiting *everything* is higher when one invests more in the project.[7] We do not claim that this is a typical description of all types of investment projects. However, it seems reasonable that large investment projects, especially of the R&D type, involve high degrees of complexity and therefore carry with them a greater probability of failure.

It is also important to note that even under Assumption A.1, the aggregate production function $F(p(K)K, \cdot) \equiv Q(K)$ can be standard, increasing and concave in K, as is usually assumed. For example, if $F(K,N) = K^{\alpha}N^{1-\alpha}$ and $p(K) = e^{-\beta K}$, with $\beta > 0$, then $Q(K) = K^{\alpha}e^{-\alpha\beta K}$, and it is easily checked that $0 < Q'(K)$ and $Q''(K) < 0$ for $K < \beta^{-1}$.[8]

For future reference, define the elasticity of success probability with respect to capital (henceforth "success elasticity") as:

$$\eta(K) \equiv \frac{Kp'(K)}{p(K)}$$

The usefulness of the following assumption on η will be readily apparent in Section 4 below.

Assumption 3 *The "success elasticity" lies in the open interval* $(-1, 0)$ *for all* $K > 0$:

$$-1 < \eta(K) \leq 0, \quad \eta(0) = 0 \tag{A.2}$$

Moreover,

$$0 < \eta'(K) < \infty \tag{A.3}$$

[6] In de Meza and Webb (1999), projects with a high "risk characteristic" yield higher return in the event of success, but have lower probability of success.

[7] The assumption that all is lost in the event of failure is stronger than what we need. We have checked that if capital depreciates less than 100%, and if undepreciated capital does not enter the $p(.)$ function, then all our subsequent results (suitably modified) go through only at the cost of additional notation.

[8] There is a more technical reason for making Assumption A.1. It is easily verified that under the opposite assumption, $p'(K) > 0$, the expected private output function $p(K)F(K,.)$ exhibits increasing returns to scale, thereby precluding a competitive environment. The assumption (A.1) is therefore crucial to our results.

Condition (A.2) of Assumption 3 implies that $1 + \eta(K_{t+1}) > 0$ for all $K > 0$, or that the success probability function is mildly elastic with respect to capital invested.[9] Condition (A.3) further strengthens the effect; the success probability function responds more positively to capital investment for higher capital stocks than for lower capital stocks.

2.2 Markets

2.2.1 Labor Market

If the project is a success, the old capitalist will go to the labor market to hire labor at the given wage rate w_{t+1}, and his firm's output is $Y_{t+1} = F(K_{t+1}, N_{t+1})$. Given K_{t+1}, the owner of the successful project will choose N_{t+1} to maximize his profit, by equating marginal product of labor to the market wage rate w_{t+1} This implies that his firm's capital labor ratio x satisfies

$$f(x_{t+1}) - f'(x_{t+1}) \cdot x_{t+1} = w_{t+1} \tag{3}$$

which implicitly defines[10]

$$x_{t+1} = \phi(w_{t+1}), \quad \phi'(.) > 0. \tag{4}$$

After making payments to labour, the successful entrepreneur gets the residual Π, where

$$\Pi \equiv f'(\phi(w_{t+1}))K_{t+1}. \tag{5}$$

In the absence of any aggregate uncertainty, each entrepreneur invests the same amount K_{t+1}, and then the measure of successful entrepreneurs is exactly $p(K_{t+1})$. After individual entrepreneurs observe their idiosyncratic random events, the economy's overall capital-labor ratio is $[p(K_{t+1})K_{t+1}/1]$ implying that

$$x_{t+1} = p(K_{t+1})K_{t+1}/1 \tag{6}$$

[9] As an example, consider the function $p(K) = \frac{\beta}{1+K}$, $\beta > 0$. Then $p'(.) < 0$, and $0 > \eta(K) = -K/(1+K) > -1$.

[10] For example, if $F(K,N) = AK^{\alpha}N^{1-\alpha}$, then it readily follows that $x_{t+1} = [w_{t+1}/A(1-\alpha)]^{1/\alpha} = \phi(w_{t+1})$ or that $w_{t+1} = A(1-\alpha)x^{\alpha}_{t+1}$.

or, equivalently,

$$p(K_{t+1})K_{t+1} = \phi(w_{t+1}) \tag{7}$$

Given the investment level K_{t+1} undertaken by all entrepreneurs, the market-clearing wage rate in period $t+1$ is then uniquely determined. Alternatively, from (7) and (3),

$$w_t = \phi'^{-1}\left[p(K_t)K_t\right] = f\left[p(K_t)K_t\right] - f'\left[p(K_t)K_t\right]\left[p(K_t)K_t\right] \equiv \psi(K_t) \tag{8}$$

It is immediate from Assumption A2 that $\psi'(K_t) > 0$. (Note that if $F(K,N) = AK^{\alpha}N^{1-\alpha}$, then it readily follows that $\psi(K_t) = (1-\alpha)\left[p(K_{t+1})K_{t+1}\right]^{\alpha}$ and $\psi'(K_t) = (1-\alpha)\left[p(K_{t+1})K_{t+1}\right]^{\alpha-1}\left[p'(K_t)K_t + p(K_{t+1})\right] > 0$.)

2.2.2 Goods and financial markets

In the scenario described above, it is apparent that financial intermediaries can arise to provide risk and consumption smoothing. In our competitive context, financial intermediaries (FIs) can be thought of as coalitions of young agents that create borrowing and lending opportunities to maximize the expected utility of the representative depositor. The FIs take advantage of the law of large numbers and lend to a large number of young capitalists. They offer a riskless interest rate (r) to depositors. The representative young individual in period t borrows B_t from the FIs, so that his available fund is $w_t + B_t$. He allocates this fund among three uses: consumption when young, C_t^y, investment in his project, K_{t+1}, and deposits in FIs, D_t. The first period budget constraint of a young capitalist is then given by

$$C_t^y = w_t + B_t - K_{t+1} - D_t.$$

The investment in the risky activity may turn out to be a success, or a failure, as has been discussed above.

We now turn to the problem of how investment decisions are made. If the risky project turns out to be a success, (i.e., the state of nature is "good") the entrepreneur will have to pay the FIs the contracted amount $R_{t+1}B_t$ where R_{t+1} is the gross rate of interest per unit on the loan. If the project is unsuccessful, (i.e., in the bad state) he will pay the FIs nothing (this reflects the limited liability feature of the loan contract). The individual's second period consumption in the "good state" is

$$C_{t+1}^{oG} = r_{t+1}D_t + f'(\phi(w_{t+1}))K_{t+1} - R_{t+1}B_t$$

(where the superscript o indicates the old age consumption, and G indicates the good state). In the "bad state", under our assumption that all investments in the risky technology are lost, and that (under limited liability) he pays the FIs nothing, we have

$$C_{t+1}^{oB} = r_{t+1}D_t.$$

Note that if $w_t, w_{t+1}, r_{t+1}, K_{t+1}$, R_{t+1} and B_t are taken as given, the consumer's choice of D_t must maximize

$$u\left[w_t + B_t - K_{t+1} - D_t\right] + \left[1 - p(K_{t+1})\right] v\left[r_{t+1}D_t\right] + p(K_{t+1})v\left[r_{t+1}D_t + f'(\phi(w_{t+1}))K_{t+1} - R_{t+1}B_t\right]$$

This maximization problem yields the function

$$D_t^* = D\left[w_t, w_{t+1}, r_{t+1}, K_{t+1}, R_{t+1}, B_t\right] \in (0, w_t)$$

which summarizes the agent's optimal deposit behavior.

3 Loan contracts

The FIs lend to the entire body of entrepreneurs, and the expected return from each unit of funds lent is $p(K_{t+1})R_{t+1}$. They also pay depositors the safe rate r_{t+1} for each unit of goods deposited in period t. Then, the zero profit condition for the FIs is given by

$$p(K_{t+1})R_{t+1} = r_{t+1}$$

from which it follows that $R_{t+1} > r_{t+1}$ must obtain. We now turn to the equilibrium determination of these returns.

The problem faced by the FIs is as follows. Suppose a FI lends a certain amount to each potential capitalist and they, in turn, invest either nothing or a tiny amount in their projects. Recall that the probability of success of any project is inversely linked to the amount invested. Under limited liability, the borrowers are protected, and therefore, the FI cannot break even. Therefore, it is evident that for FIs to survive, they must be able to induce the borrower to invest the "right" amount. Under our assumption of complete transparency and perfect information, this requires that the FIs must have the ability to perfectly monitor, observe, and control the actions of the borrowers.

We now proceed to derive the optimal loan contract that FIs offer to the entrepreneurs under the assumption of complete control. In other words, the FI that lends to an entrepreneur actively monitors him (in the sense of Holmstrom and Tirole, 1997), and *observes and dictates* the amount K_{t+1} that he invests in the risky project. As is well-known, the efficient contract must maximize the expected utility of the representative borrower, given the limited liability constraint, and the "participation constraint" of the FI.

Formally, given w_t, w_{t+1}, r_{t+1}, the design of the efficient contract solves the following problem: choose D_t, K_{t+1}, B_t, R_{t+1} to maximize an entrepreneur's expected utility

$$U = u\left[w_t + B_t - K_{t+1} - D_t\right] + \left[1 - p(K_{t+1})\right] v\left[r_{t+1}D_t\right] + p(K_{t+1}) v\left[r_{t+1}D_t + f'(\phi(w_{t+1}))K_{t+1} - R_{t+1}B_t\right]$$

subject to the lender's participation constraint:

$$p(K_{t+1})R_{t+1} = r_{t+1} \tag{9}$$

Forming the Lagrangian,

$$£ = U + \lambda\left[p(K_{t+1})R_{t+1} - r_{t+1}\right]$$

The first order necessary conditions for an interior solution are[11]:

(i) w.r.t. D_t

$$-u'(C_t^y) + r_{t+1}p(\cdot)\, v'\left[C_{t+1}^{oG}\right] + r_{t+1}(1 - p(\cdot))v'\left[C_{t+1}^{oB}\right] = 0 \tag{10}$$

(ii) w.r.t. K_{t+1}

$$-u'(C_t^y) + p'(\cdot)\, v\left[C_{t+1}^{oG}\right] + p(\cdot)\, v'\left[C_{t+1}^{oG}\right] f'(\phi(w_{t+1})) - p'(\cdot)\, v\left[C_{t+1}^{oB}\right] + \lambda p'(\cdot)\, R_{t+1} = 0 \tag{11}$$

(iii) w.r.t. B_t

$$u_1'(C_t^y) - R_{t+1}p(\cdot)\, v'\left[C_{t+1}^{oG}\right] = 0 \tag{12}$$

(iv) w.r.t. R_{t+1}

$$p(\cdot)\, v'\left[C_{t+1}^{oG}\right] B_t - \lambda p(\cdot) = 0 \tag{13}$$

From (10) and (12), it follows that

$$r_{t+1}p(\cdot)\, v'\left[C_{t+1}^{oG}\right] + r_{t+1}(1 - p(\cdot))v'\left[C_{t+1}^{oB}\right] = R_{t+1}p(\cdot)\, v'\left[C_{t+1}^{oG}\right]$$

From (13), a positive B_t implies $\lambda > 0$. Thus, (9) is binding. Then, $R_{t+1}p(\cdot) = r_{t+1}$ holds, implying

$$p(\cdot)\, v'\left[C_{t+1}^{oG}\right] + (1 - p(\cdot))v'\left[C_{t+1}^{oB}\right] = v'\left[C_{t+1}^{oG}\right]$$

or,

$$p(\cdot)\left\{v'\left[C_{t+1}^{oG}\right] - v'\left[C_{t+1}^{oB}\right]\right\} = \left\{v'\left[C_{t+1}^{oG}\right] - v'\left[C_{t+1}^{oB}\right]\right\}$$

Since $p(\cdot) < 1$, this implies

$$C_{t+1}^{oG} = C_{t+1}^{oB}. \tag{14}$$

In other words, second period consumption is independent of the state of nature. This is a standard result: the risk-neutral FI provides complete insurance to the risk-averse consumer. An immediate

[11] As is true with similar problems in much of this literature, it is not immediate that the function $U(B, K, D, R)$ is strictly concave in its arguments.

consequence of eq. (14) is that the ratio of the loan to the amount of capital invested in the risky project is equal to the ratio of (ex-post) marginal product of capital to the gross interest rate on the risky loan:

$$\frac{B_t}{K_{t+1}} = \frac{f'(\phi(w_{t+1}))}{R_{t+1}} \tag{15}$$

It remains for us is to describe the relationship between r_{t+1} and K_{t+1}, summarized in the next lemma.

Lemma 1 *Given w_{t+1} and the safe rate of return r_{t+1}, the efficient contract specifies a unique investment amount K_{t+1}. This amount equates the expected rate of return, modified by the success elasticity, to the safe rate of return:*

$$r_{t+1} \equiv r(K_{t+1}) = p(K_{t+1})f'(\phi(w_{t+1}))\,[1+\eta(K_{t+1})] \tag{16}$$

Under condition (A.2) of Assumption 3, it follows from (16) that the safe rate of return is *lower* than the expected rate of return from investing in capital. From the zero-profit condition, eq. (9), it follows that

$$R_{t+1} = f'(\phi(w_{t+1}))\,[1+\eta(K_{t+1})]$$

which using (15) yields

$$\frac{K_{t+1}}{B_t} = 1+\eta(K_{t+1}) \Leftrightarrow B_t > K_{t+1} \forall\ K_t > 0. \tag{17}$$

The fact that the efficient contract requires the intermediary to disburse a loan of size greater than the amount the borrower invests (in the risky capital project) is important. The underlying reason is that since the borrower is risk-averse and might forfeit all of his investment in the event of a failure, he will not invest the entire amount of the loan in the capital technology.

Condition (17) implies that B_t can be expressed as a function of K_{t+1}:

$$B_t = B_t(K_{t+1}) = \frac{K_{t+1}}{1+\eta(K_{t+1})} \tag{18}$$

Define

$$Z_t \equiv B_t(K_{t+1}) - K_{t+1} \equiv Z(K_{t+1}) \tag{19}$$

as the excess of the loan over the capital investment. The variable (Z_t) plays a key role in our set-up because it captures the "income effect" that is ultimately responsible for generating endogenous fluctuations in our economy.

In passing, we quickly establish certain useful properties of the optimal safe deposits D. Using (19) and (16), we can simplify condition (10) to read

$$-u'\left[w_t + Z_t - D_t^*\right] + r_{t+1}v'\left[r_{t+1}D_t^*\right] = 0$$

which implicitly defines

$$D_t^* = D\left[w_t, r_{t+1}, Z_t\right].$$

Straightforward differentiation reveals that

$$D_w \equiv \frac{\partial D}{\partial w_t} = \frac{1}{\Delta}u'' > 0, \qquad \Delta \equiv u'' + (r_{t+1})^2 v'' < 0,$$

$$D_r \equiv \frac{\partial D}{\partial r_t} = -\frac{1}{\Delta}\left[v' + C_{t+1}^o v''\right] \geq 0,$$

because of Assumption 1, and finally,

$$D_z \equiv \frac{\partial D}{\partial Z_t} = \frac{1}{\Delta}u'' > 0.$$

This last result ties in well with our earlier observation that the excess of the loan amount over the capital invested shows up in higher investment by the agent in the safe asset.

4 General Equilibrium

We require that the loan market be in equilibrium, therefore the deposits D_t with the FIs must be equal to the loans the FIs make to entrepreneurs:

$$D\left[w_t, r_{t+1}, Z_t\right] = B_t \tag{20}$$

Using (8) and (16), (20) may be written as

$$D\left[\psi(K_t), r(K_{t+1}), Z(K_{t+1})\right] = B_t(K_{t+1}) \tag{21}$$

This first-order, (potentially) non-linear difference equation summarizes all the competitive dynamical equilibria in the economy. Given Assumptions 1-3, starting from any $K_1 > 0$, equation (21) generates a solution sequence $\{K_t\}$ from which every endogenous variable in the model can be computed. We now proceed to study equation (21) in some detail, starting with a study of steady states.

4.1 Stationary equilibria

Stationary equilibria are time-invariant sequences $\{K\}$ that satisfy (21). Straightforward differentiation of (18) reveals that the right hand side of (21) may be upward or downward sloping, or that

$$B'(K) = \frac{1}{\left[1+\eta(K)\right]^2}\left[1+\eta\left(\cdot\right) - K\eta'\left(K\right)\right] \gtreqless 0 \tag{22}$$

where, recall that (A.3) states that $\eta'\left(K\right) > 0$. Similarly, recall that $\psi'(K) = \frac{1}{\phi'}\left[p'(K)K + p(K)\right] > 0$. The slope of the left hand side of (21) is given by

$$D_w\psi'(K) + D_r r'(K) + D_z\left[B'(K) - 1\right].$$

where $D_r \geq 0$ by Assumption 1. We note that $B'(K) \lesseqgtr 0$ can obtain, and thus the left hand side of (21), much like the right hand side, may be upward or downward sloping. In other words, the relationship between the size of the loan and the stipulated capital investment may be non-monotonic as a consequence of Assumption A.3. This is true *even* if $D_r = 0$. If $u'(0) = v'(0) = \infty$, then $K = 0$ is definitely a steady state. (Note that $B(0) = 0$, $\psi(0) = 0$, and $Z(0) = 0$).

Example 1 *Let $u = \ln C^y$ and $v = \ln C^o$. Then (21) reduces to*

$$\psi(K) = \left[\frac{2+\eta\left(\cdot\right)}{1+\eta\left(\cdot\right)}\right]K. \tag{23}$$

Choose $f(p(K)K) = [p(K)K]^{\alpha}$, *and set* $\alpha = 0.36$. *Also, choose* $p(K) = 1/[1 + \ln(1 + K)]$. *It is easy to verify that Assumptions (A.1)-(A.3) are met. Then (23) has a unique non-trivial steady state,* $K^* = 0.1429$.

It is quite possible that there are multiple interior steady states, just as in the standard Diamond (1965) model. We now proceed to study the dynamics of the model, paying special attention to the possibility of existence of cyclical equilibria.

4.2 Dynamic equilibria

Using (21), we can obtain the slope as

$$\frac{dK_{t+1}}{dK_t} = \frac{-D_w \psi'(K_t)}{D_Z\left[B'(K_{t+1}) - 1\right] + D_r r'(K_{t+1}) - B'(K_{t+1})} \tag{24}$$

Clearly, the numerator of the right-hand side of (24) is negative. The denominator is ambiguous in sign, because $B'(.)$ and $r'(.)$ are of ambiguous sign under Assumption 2.[12] It bears emphasis here that even if $D_r = 0$, the denominator of the right-hand side of (24) is ambiguous in sign. It follows that there may exist points at which the denominator of (24) changes sign, i.e., the locus of points (K_t, K_{t+1}) that satisfy (21) may well be a correspondence $K_t = \chi(K_{t+1})$: for a given K_{t+1}, there may exist more than one value of K_t but for a given K_t, there is only one value of K_{t+1}. Then, we can implicitly define a function $K_{t+1} = G(K_t)$ representing the standard backward dynamics.

Lemma 2 *The slope of the locus* $K_{t+1} = G(K_t)$ *changes sign only at a point* $\hat{K}$ *where*

$$D_Z\left[B'(\hat{K}) - 1\right] + D_r r'(\hat{K}) - B'(\hat{K}) = 0.$$

[12] Since we have assumed that, for positive K, $1 > 1 + \eta > 0$, it follows that B is always positive and greater than K for all $K > 0$. Thus it is not possible to have $B'(K_{t+1}) < 0$ for all K; nevertheless, it is possible that $B'(K_{t+1}) < 0$ over some intervals where η' is positive and sufficiently large.

Suppose $D_r = 0$ holds, then $\hat{K}$ satisfies

$$B'(\hat{K}) = -\frac{u''(\cdot)}{\left[r(\hat{K})\right]^2 v''(\cdot)} < 0$$

As is well-known (see Azariadis, 1993, Chapter 8), a necessary (but not sufficient) condition for the map G to exhibit cyclical fluctuations in the standard Diamond model is $D_r < 0$. Our goal in this paper is to prove the possibility of cyclical equilibria in an otherwise Diamond-like economy, augmented only by an active financial intermediation sector. To that end, our claim that our model generates periodic equilibria is best supported by "tying our hands behind our back" and focusing on examples where in fact $D_r = 0$ holds.

4.2.1 An example with logarithmic preferences and Cobb Douglas technology

Henceforth, we focus exclusively on analyzing the prototype logarithmic utility case. If $u = \ln C^y$ and $v = \ln C^o$, then it is easily checked that the difference equation (21) simplifies to

$$\psi(K_t) = \left[\frac{2+\eta(\cdot)}{1+\eta(\cdot)}\right] K_{t+1} \equiv \gamma(K_{t+1}) \tag{25}$$

For future reference, we define $G(.) \equiv \gamma^{-1}(\psi(\cdot))$, implying once again that $K_{t+1} = G(K_t)$. If $p(K)K$ approaches zero as K tends to zero, and if w goes to zero as x tends to zero, then (25) has $(0,0)$ as a stationary point. Furthermore, from (25) it follows that

$$\frac{dK_{t+1}}{dK_t} = \frac{\psi'(K_t)}{\dfrac{2+\eta(\cdot)}{1+\eta(\cdot)} - \dfrac{\eta'(\cdot)\,K_{t+1}}{[1+\eta(\cdot)]^2}} \tag{26}$$

Recall that, for the Cobb-Douglas production function, under (A.3),

$$\psi'(K_t) = (1-\alpha)\left[p(K_{t+1})K_{t+1}\right]^{\alpha-1}\left[p'(K_t)K_t + p(K_{t+1})\right] > 0$$

holds. As stated earlier, a necessary (but not sufficient) condition for periodic equilibria (including two-period cycles) is that $\frac{dK_{t+1}}{dK_t}$ be negative (locally near the steady state); a necessary (but not sufficient) condition for complex dynamics (cycles of higher order, and even aperiodic equilibria) is that $\frac{dK_{t+1}}{dK_t}$ change sign.[13] These aforementioned necessary conditions therefore reduce to the condition that the denominator of (26) be negative or change sign.

It follows from (26) that $\frac{dK_{t+1}}{dK_t}$ changes sign only once if and only if there exists a unique value $\hat{K} > 0$ that satisfies

$$\left[1+\eta(\hat{K})\right]\left[2+\eta(\hat{K})\right] = \hat{K}\eta'(\hat{K}) \tag{27}$$

Under (A.3) and (A.2), the left hand side of (27) starts off at 2 [with slope $3\eta'(\cdot)$] and is non-decreasing in K. Define $\varepsilon \equiv \frac{K\eta''(K)}{\eta'(K)}$. Then, clearly a sufficient condition for (27) to yield a unique $\hat{K}$ is that the slope of the right hand side of (27) be always greater than the slope of the left hand side, or that

$$\varepsilon > 2\left[1+\eta(\cdot)\right] \quad \forall K > 0. \tag{A.4}$$

Under (A.4), the map G has a possible configuration of the type illustrated in Figure 1f. In this case, there are exactly two steady states, K_a^* and K_b^* with $K_a^* < \hat{K} < K_b^*$. Next, we compute a precise example of an economy which satisfies Assumptions (1)-(3) and also exhibits a cycle of periodicity 2.

****************** **FIGURES 1 - 6 HERE** ***************************

Example 2 *(Two-period cycle) Let $K > 7$ and choose*

$$p(K) = \frac{0.32e^K}{1.2 + 1.901885e^K - K^3}.$$

Choose $f(p(K)K) = 28\left[p(K)K\right]^{0.33}$. From the loci for $p(.)$, $\psi(.)$, $\eta(\cdot)$, and $\eta'(\cdot)$, drawn in Figures 1-4, it is clear that Assumptions (1)-(3) are verified. Then, the equation $\psi(K_t) = \gamma(K_{t+1})$, defined in (25), and illustrated in Figure 5, has two stationary solutions, $K_a^* = 8.94$ and $K_b^* = 10.89$. Additionally, $\frac{dK_{t+1}}{dK_t}|_{K=K_a^*} = -1.00$ implying that there exists a stable two-period cycle in a neighborhood around K_a^*, as illustrated in Figure 6.

[13] See Appendix C for a technical discussion of periodic equilibria.

A few words on the underlying nature of these two-period cycles are in order. For clarity of exposition, consider an economy where agents care only about second-period consumption. Focus on a starting situation where the capital stock in the economy is relatively high. Then, the wage rate in the economy is high, and so are saving and desired investment as fractions of wage income. Left unfettered, if actual investment in the risky capital technology were high, the riskiness of that technology would be very high, and overall expected return would be very low. The bank, recognizing the problem, steps in and disburses a large loan and dictates a low *actual* investment so as to keep the riskiness low, and expected return high.[14] So, the amount invested in the capital technology is low and the amount invested in the safe deposits with the bank is high. The output in the following period is then low (given the low actual investment). Then next period's wage rate is low, and so are saving and desired investment as fractions of wage income. In the absence of the bank, a low wage income would imply a low desired investment and hence low riskiness but low expected return. Under our assumptions on the success probability function, the bank may be able to avoid this bad situation by disbursing a small loan and stipulating a high actual investment. The high investment may raise the expected return to investing in the capital technology, and raise output next period, and so on.

Under our assumptions on the success probability function, the size of the loan relative to what the bank stipulates as capital investment varies with capital. This difference is what we have labeled the "income effect". We have shown that the strength of this income effect can either be decreasing or increasing with the bank's stipulation of capital investment. This is because of the assumed effect

[14] To assess the empirical plausibility of such phenomena, at least as a first pass, one can study whether the correlation between private investment and the premium of commerical paper over a safe T-bill rate is negative. Indeed, for the United States and for the period 1986-1997, the correlation between the premium of 3-month commercial paper rate over the 3-month Treasury constant maturity rate and Gross Private Domestic Investment (all at the quarterly frequency) was -0.11. The data was taken from the FRED-II website [http://research.stlouisfed.org/fred2/] of the Federal Reserve Bank of St. Louis. We thank the editor for suggesting the above exercise.

of capital on the success probability. If capital invested is high, success probability is low and, under our assumptions on elasticities, expected return is falling for low K and after a threshold, starts to rise with K. Hence, the non-monotone income effect, and hence the cycles in special circumstances.

An interesting possibility that arises, especially in inverted-U shaped maps, is that the low capital stock steady state can be locally stable. In other words, economies that start off in a neighborhood of this low-activity steady state may find themselves "stuck" in a development trap. Such paths will display damped endogenous oscillations along the way. The next example contains such a development trap.

Example 3 *(Development Trap) Let $K > 5$ and choose*

$$p(K) = \frac{0.1e^K}{1.5 + 1.5e^K - K^{2.5}}.$$

Choose $f(p(K)K) = 28\,[p(K)K]^{0.2}$. As before, it can be verified that Assumptions (1)-(3) are verified. Then, the map G defined in (25) has two stationary solutions, $K_a^* = 6.27$ and $K_b^* = 9.9$. Additionally, $\frac{dK_{t+1}}{dK_t}|_{K=K_a^*} = -0.11$ implying that orbits that start off in a local neighborhood around K_a^* experience damped oscillations but eventually converge to it.

The implication of the above example is clear. Consider two economies, identical in every respect, except for the size of their initial capital stock. Then, one of them converges to a low-level of real activity and cannot escape from there. Moreover, it encounters endogenous oscillations, albeit damped ones, along the path to that trap. The other one approaches a high-level of real activity but cannot sustain it indefinitely.

The possibility arises that some dynamical equilibria may exhibit *undamped* oscillations. To see this, consider the particular configuration of $G(\cdot)$ as illustrated in Figure 7. Define $\breve{K}$ by $G'(\breve{K}) = 0$ and $\tilde{K}$ by $G(\tilde{K}) = G(K_a^*)$ with $G'(\tilde{K}) < 0$.

Proposition 1 *Suppose K_a^* is a repelling point and $\tilde{K} > G(\breve{K})$. Then, in every neighborhood of K_a^*, there are infinitely many distinct periodic points; indeed, equilibrium cycles of periodicity 2^i exists for* $i = 1, 2, 3...$

The condition $\tilde{K} > G(\breve{K})$ implies the existence of a homoclinic orbit or that K_a^* is a snap-back repellor in the sense of Devaney (1989, pg. 122). An additional variety of non-convergence can now be observed. Consider two otherwise identical economies. Depending on the initial capital stock, one may approach K_b^* and the other may get infected by cycles of all even order all along the path to K_a^*, approaching it but never getting permanently stuck at the low level of real activity at K_a^*.

******************* **FIGURES 7-8 HERE** ****************************

Example 4 *(Cycles of all even order) Let $K > 5$ and choose*

$$p(K) = \frac{0.25e^K}{1 + 3e^K - K^3}.$$

Choose $f(p(K)K) = 33\,[p(K)K]^{0.33}$. As before, it can be verified that Assumptions (A.1)-(A.3) are verified. Then, the map G defined in (25) has two stationary solutions, $K_a^* = 7.61$ and $K_b^* = 9.58$. Additionally, $\frac{dK_{t+1}}{dK_t}|_{K=K_a^*} = -1.58$ implying that the low capital stock steady state is a repellor. This economy possesses a homoclinic orbit. Then, in every neighborhood of K_a^*, there are infinitely many distinct periodic points.

Finally, it is possible that our economy generates an equilibrium displaying a three-period cycle, as illustrated in Figure 8. If that is the case, then from the well-known Li-Yorke theorem, as discussed in Azariadis (1993, Ch. 9) and Appendix C, it is clear that cycles of all order, even or odd exist; additionally aperiodic or chaotic equilibria also exist. In short, if the configuration in Figure 8 is achieved, then the existence of topological chaos is established. This last observation creates a third source of non-convergence in our model. Two economies that are intrinsically identical may "look alike" initially and even for some time, but will never "look the same" eventually.

4.3 Absence of financial intermediaries

In the previous sub-section, we have demonstrated the real possibility that the presence of financial intermediaries, specifically their tight control over capital investment, opens the door for development traps and a variety of complex dynamics, including high-order cycles. In order to clinch this argument, we now undertake a quick study of the dynamical equilibria in an environment identical to the one studied above, except that, for exogenous reasons, financial intermediaries are totally absent. In such a situation, risk-averse individuals simply decide how much to invest in the risky capital technology. Also, there are no safe assets.

The representative entrepreneur chooses K_{t+1} to maximize his expected utility:

$$EU = u_1 \left[C_t^y\right] + p\left(K_{t+1}\right) v_2 \left[C_{t+1}^{oG}\right] + \left[1 - p\left(K_{t+1}\right)\right] v_2 \left[C_{t+1}^{oB}\right] \tag{28}$$

where

$$C_t^y = w_t - K_{t+1}, \quad C_{t+1}^{oG} = \rho_{t+1} K_{t+1}, \quad C_{t+1}^{oB} = 0 \tag{29}$$

Agents takes the gross real return on capital (ρ) and the wage rate as given. Then,

$$EU = u_1 \left[C_t^y\right] + p\left(K_{t+1}\right) v_2 \left[C_{t+1}^{oG}\right]$$

The first order necessary conditions with respect to K_{t+1} for a solution in the interior are given by

$$u_1' \left[C_t^y(\theta)\right] = p'\left(K_{t+1}\right) v_2 \left[C_{t+1}^{oG}\right] + p\left(K_{t+1}\right) v_2' \left[C_{t+1}^{oG}\right] \left[\rho_{t+1}\right] \tag{30}$$

In a general equilibrium, from the standard factor pricing relationships described earlier, we have

$$\rho_{t+1} = f'(x_{t+1}), \quad w_t = w\left(x_t\right)$$

where $x_t \equiv p(K_t)K_t$.Then (30) implies

$$u_1' \left[w\left(x_t\right) - K_{t+1}\right] = p'\left(K_{t+1}\right) v_2 \left[f'(x_{t+1}) \cdot K_{t+1}\right] + p\left(K_{t+1}\right) v_2' \left[f'(x_{t+1}) \cdot K_{t+1}\right] \left[f'(x_{t+1})\right] \tag{31}$$

Proposition 2 *Suppose Assumptions 1,2 and 3 hold, and that in addition,* $p''(.) < 0$ *holds. Then, for the case of Cobb-Douglas technology,* $f(K) = AK^{\alpha}$, $\alpha \in (0,1)$, *there is no possibility for cycles in the economy with no banks (and no safe asset).*

In other words, (31) implicitly defines a monotonic relationship between the capital stock this period and the next, thereby precluding the possibility of any endogenous fluctuations. The upshot is that it has to be the activities of the banks (especially the fact that the amount they dictate be invested in capital is less than the size of the loan they offer) is what is fundamentally responsible for the output volatility.

5 Conclusion

This paper focuses on the possibility of endogenous fluctuations caused by activities of financial intermediaries within the context of a simple overlapping generations model. Risk-averse agents face idiosyncratic income losses, the probability of which they can affect through their own capital investments. We showed that in the economy with intermediation under full observability, the optimal loan contract achieves complete risk sharing but the amount invested in the risky project is smaller than the loan offered. This last fact alone creates an income effect which is responsible for the endogenous generation of cycles and complex dynamics. The analysis indicated that in the absence of any intermediation, the economy studied would not exhibit any fluctuations of any kind.

In the set-up studied above, problems of moral hazard and adverse selection were assumed away. Doubtless these omissions are important departures from reality, and should be addressed by future research. Another worthwhile extension would be to study alternative formulations of the probability function, including those in which not all is lost in the event of failure.

Appendix

A Proof of Lemma 1

Using (14), (11), (12) and (13), we get

$$r_{t+1} = pf'(\phi(w_{t+1})) + B_t R_{t+1} p'(\cdot) \tag{32}$$

Using (15) in (32), we get

$$r_{t+1} = pf'(\cdot) + Kp'(\cdot) f'(\cdot) \tag{33}$$

which immediately yields (16).■

B Proof of Proposition 2

For future reference,

$$\begin{aligned}
\frac{\partial x_t}{\partial K_t} &= \left[p'(K_t)K_t + p(K_t)\right] \\
\frac{\partial x_{t+1}}{\partial K_t} &= \left[p'(K_{t+1})K_{t+1} + p(K_{t+1})\right] \frac{\partial K_{t+1}}{\partial K_t}
\end{aligned}$$

Recall from Assumption 3 that

$$\frac{Kp'(K)}{p(K)} + 1 > 0$$

Then,

$$\begin{aligned}
\frac{\partial x_t}{\partial K_t} &= \left[p'(K_t)K_t + p(K_t)\right] = p(K)\left[\frac{Kp'(K)}{p(K)} + 1\right] > 0 \\
\frac{\partial x_{t+1}}{\partial K_t} &= \left[p'(K_{t+1})K_{t+1} + p(K_{t+1})\right] \frac{\partial K_{t+1}}{\partial K_t} = p(K)\left[\frac{Kp'(K)}{p(K)} + 1\right] \frac{\partial K_{t+1}}{\partial K_t}
\end{aligned}$$

Then, it follows from (31) that

$$u_1''(\cdot)\left[w'(x_t)\frac{\partial x_t}{\partial K_t} - \frac{dK_{t+1}}{dK_t}\right]$$

$$= p''(K_{t+1})\, v_2(\cdot)\frac{dK_{t+1}}{dK_t} + p'(K_{t+1})\, v_2'(\cdot)\left[f''(x_{t+1})\frac{\partial x_{t+1}}{\partial K_t}K_{t+1} + f'(x_{t+1})\frac{dK_{t+1}}{dK_t}\right]$$

$$+p'(K_{t+1})\frac{\partial K_{t+1}}{\partial K_t}v_2'(\cdot)\, f'(x_{t+1}) + p(K_{t+1})\, f'(x_{t+1})v_2''(\cdot)\left[f''(x_{t+1})\frac{\partial x_{t+1}}{\partial K_t}K_{t+1} + f'(x_{t+1})\frac{dK_{t+1}}{dK_t}\right]$$

$$+p(K_{t+1})\, v_2'(\cdot)\, f''(x_{t+1})\frac{\partial x_{t+1}}{\partial K_t}$$

Expanding, we get

$$u_1''(\cdot)\,w'(x_t)\,\frac{\partial x_t}{\partial K_t}-u_1''(\cdot)\,\frac{dK_{t+1}}{dK_t}$$

$$=p''(K_{t+1})\,v_2(\cdot)\,\frac{dK_{t+1}}{dK_t}+p'(K_{t+1})\,v_2'(\cdot)\left[f''(x_{t+1})\left\{p'(K_{t+1})K_{t+1}+p(K_{t+1})\right\}K_{t+1}+f'(x_{t+1})\right]\frac{dK_{t+1}}{dK_t}$$

$$+p'(K_{t+1})\,\frac{\partial K_{t+1}}{\partial K_t}v_2'(\cdot)\,f'(x_{t+1})+p(K_{t+1})\,f'(x_{t+1})v_2''(\cdot)\left[f''(x_{t+1})\left\{p'(K_{t+1})K_{t+1}+p(K_{t+1})\right\}K_{t+1}+f'(x_{t+1})\right]\frac{dK_{t+1}}{dK_t}$$

$$+p(K_{t+1})\,v_2'(\cdot)\,f''(x_{t+1})\left[p'(K_{t+1})K_{t+1}+p(K_{t+1})\right]\frac{\partial K_{t+1}}{\partial K_t}$$

Then, rearranging terms yields

$$\frac{dK_{t+1}}{dK_t}=\frac{u_1''(\cdot)\,w'(\cdot)\,\frac{\partial x_t}{\partial K_t}<0}{\begin{array}{c}p''(\cdot)\,v_2(\cdot)+p'(\cdot)\,v_2'(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1}+p(\cdot)\right\}K_{t+1}+f'(\cdot)\right]+p'(\cdot)\,v_2'(\cdot)\,f'(\cdot)+\\ p(\cdot)\,f'(\cdot)v_2''(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1}+p(\cdot)\right\}K_{t+1}+f'(\cdot)\right]+p(\cdot)\,v_2'(\cdot)\,f''(\cdot)\left[p'(\cdot)K_{t+1}+p(\cdot)\right]+u_1''(\cdot)\end{array}}$$

Focus on the denominator from now on. We find that

$$p''(\cdot)\,v_2(\cdot)+p'(\cdot)\,v_2'(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1}+p(\cdot)\right\}K_{t+1}+f'(\cdot)\right]+\underbrace{p'(\cdot)\,v_2'(\cdot)\,f'(\cdot)}_{<0}$$
$$+p(\cdot)\,f'(\cdot)v_2''(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1}+p(\cdot)\right\}K_{t+1}+f'(\cdot)\right]+\underbrace{p(\cdot)\,v_2'(\cdot)\,f''(\cdot)\left[p'(\cdot)K_{t+1}+p(\cdot)\right]}_{<0}+\underbrace{u_1''(\cdot)}_{<0}$$

It follows that if $[f''(\cdot)\{p'(\cdot)K_{t+1}+p(\cdot)\}K_{t+1}+f'(\cdot)]>0$ and $p''<0$, the denominator would be negative. For Cobb-Douglas technology, we have

$$\frac{f''(\cdot)K}{f'(\cdot)}=\frac{K\cdot A\alpha\,(\alpha-1)\,K^{\alpha-2}}{A\alpha K^{\alpha-1}}=(\alpha-1)\,.$$

Then it is easy to check that

$$\left[f''(\cdot)\left\{p'(\cdot)K_{t+1}+p(\cdot)\right\}K_{t+1}+f'(\cdot)\right]$$
$$=\quad f''(\cdot)K_{t+1}\left[\underbrace{p(\cdot)\left\{\frac{p'(\cdot)K_{t+1}}{p(\cdot)}+1\right\}}_{>0}+\underbrace{\left(\frac{1}{\alpha-1}\right)}_{<0}\right]\gtreqless 0$$

Recall from Assumption 3 that $-1<\eta(K)\leq 0$. Then, it follows that

$$f''(\cdot)K_{t+1}\left[\underbrace{p(\cdot)\left\{\underbrace{\eta(K)+1}_{<1}\right\}}_{<1}-\underbrace{\left(\frac{1}{1-\alpha}\right)}_{>1}\right]>0$$

Then, the denominator is given by

$$p''(\cdot)\,v_2(\cdot) + \underbrace{p'(\cdot)\,v_2'(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1} + p(\cdot)\right\}K_{t+1} + f'(\cdot)\right]}_{<0} + \underbrace{p'(\cdot)\,v_2'(\cdot)\,f'(\cdot)}_{<0}$$
$$+\underbrace{p(\cdot)\,f'(\cdot)v_2''(\cdot)\left[f''(\cdot)\left\{p'(\cdot)K_{t+1} + p(\cdot)\right\}K_{t+1} + f'(\cdot)\right]}_{<0} + \underbrace{p(\cdot)\,v_2'(\cdot)\,f''(\cdot)\left[p'(\cdot)K_{t+1} + p(\cdot)\right]}_{<0} + \underbrace{u_1''(\cdot)}_{<0}$$

Then, if $p'' < 0$, $\frac{dK_{t+1}}{dK_t} > 0$ will hold, and there will be no possibility of any cyclical equilibria.

C Periodic and Chaotic Equilibria

Let X be a closed and convex subset of the real number line. Let G be a continuous function that maps X into X. The pair (X, G) is called a dynamic system. Let G^n denote the *nth* iterate of G. A point k^* is called a periodic point of (X, G) with period m (where m is an integer), if $G^m(k^*) = k^*$ but $G^n(k^*) \neq k^*$ for $n = 1, 2, ..., m-1$.The point k^* is then said to generate period-m cycles.

A subset Y of X is called a *scrambled set* of the dynamic system (X, G) if Y has the following properties:

(a) Y has an uncountable number of points,

(b) Y does not contain any periodic points of the dynamic system (X, G),

(c) for any $u, v \in Y$, where $u \neq v$,

$$\lim_{i\to\infty} \sup \|G^i(u) - G^i(v)\| > 0 \text{ and } \lim_{i\to\infty} \inf \|G^i(u) - G^i(v)\| = 0,$$

(d) for any periodic point $k^* \in X$ and any $u \in Y$

$$\lim_{i\to\infty} \sup \|G^i(u) - G^i(k^*)\| > 0.$$

If a dynamic system (X, G) has a scrambled set, we say that the dynamic system is *chaotic.*

Property (d) means that the orbit generated by any point in the scrambled set does not converge to a limit cycle. Property (c) means that any two points from the scrambled set will generate two paths that will eventually get very close together in one sense, yet remain far apart, in another sense.

The following theorem by Li and Yorke (see Rasband, 1990) states the connection between chaos and the existence of period 3 cycles:

Theorem: If there is a point $x \in X$ such that

$$G^3(x) \le x < G(x) < G^2(x) \text{ or } G^3(x) \ge x > G(x) > G^2(x) \tag{34}$$

then (i) for every positive integer m, there is a periodic point of period m, (ii) there is a scrambled set Y in X.

References

[1] Aghion, P., Banerjee, A., Piketty, T., 1999. Dualism and macroeconomic volatility. Quarterly Journal of Economics CXIV (4), 1359-1399.

[2] Allen, F and Gale, D., 1997. Financial markets, intermediaries and intertemporal smoothing. Journal of Political Economy 105, 523-546.

[3] Azariadis, C., 1981. Self-fulfilling prophecies. Journal of Economic Theory 25, 380-96.

[4] — 1993. Intertemporal Macroeconomics, Basil Blackwell: Cambridge.

[5] — , Smith B.D,. 1996. Financial intermediation and regime switching in business cycles. American Economic Review 88(3), 516-536.

[6] Bachetta, P., Caminal, R., 2000. Do capital market imperfections exacerbate output fluctuations?. European Economic Review 44, 449-468.

[7] Bernanke, B., and Gertler, M. 1989. Agency costs, net worth, and business fluctuations. American Economic Review 79(1), 14-31.

[8] Boldrin, M., Woodford, M. 1990. Equilibrium models displaying endogenous fluctuations and chaos: a survey. Journal of Monetary Economics 25, 189-222.

[9] Den Haan, W.J., G. Ramey, J. Watson (2003). Fragility and business enterprises. Journal of Monetary Economics 50(6), 215-1241.

[10] Devaney , R.L (1986), An Introduction to Chaotic Dynamical Systems. Addison-Wesley: New York

[11] de Meza, D., Webb, D. 1999. Wealth, enterprise and credit policy. Economic Journal 109, 153-63.

[12] Diamond, P. 1965. National debt in a neoclassical growth model. American Economic Review 55 (5), 1126-1150

[13] Friedman, M. 1960. A Program for Monetary Stability. Fordham University Press: New York

[14] — and Schwartz, A.J. 1963. A Monetary History of the United States, 1867-1960. Princeton, NJ: Princeton University Press

[15] Grandmont, J.M. 1985. On endogenous competitive business cycles. Econometrica 53, 995-1045

[16] Holmstrom, B., Tirole, J. 1997. Financial intermediation, loanable funds, and the real sector. Quarterly Journal of Economics 112(3), 663-691.

[17] Kaminsky, G. L., Reinhart, C.M. 1998. Financial crises in Asia and Latin America: then and now. American Economic Review Papers and Proceedings 88(2), 444-448.

[18] Pagano, M. 1993. Financial markets and economic growth: an overview. European Economic Review 37, 613-622.

[19] Rasband, N S. 1990. Chaotic Dynamics of Non-Linear Systems. Wiley, New York.

[20] Sargent, T., Wallace, N.1982. The real bills doctrine vs. the quantity theory: a reconsideration. Journal of Political Economy 90, 1212–1236.

[21] Smith, B. D. 1988. Legal restrictions, "sunspots," and Peel's Bank Act: the real bills doctrine versus the quantity theory reconsidered. Journal of Political Economy, 96 (1), 3-19.

[22] Williamson, S.D. 1987. Financial intermediation, business failures, and real business cycles. Journal of Political Economy 95(6), 1196- 1216.

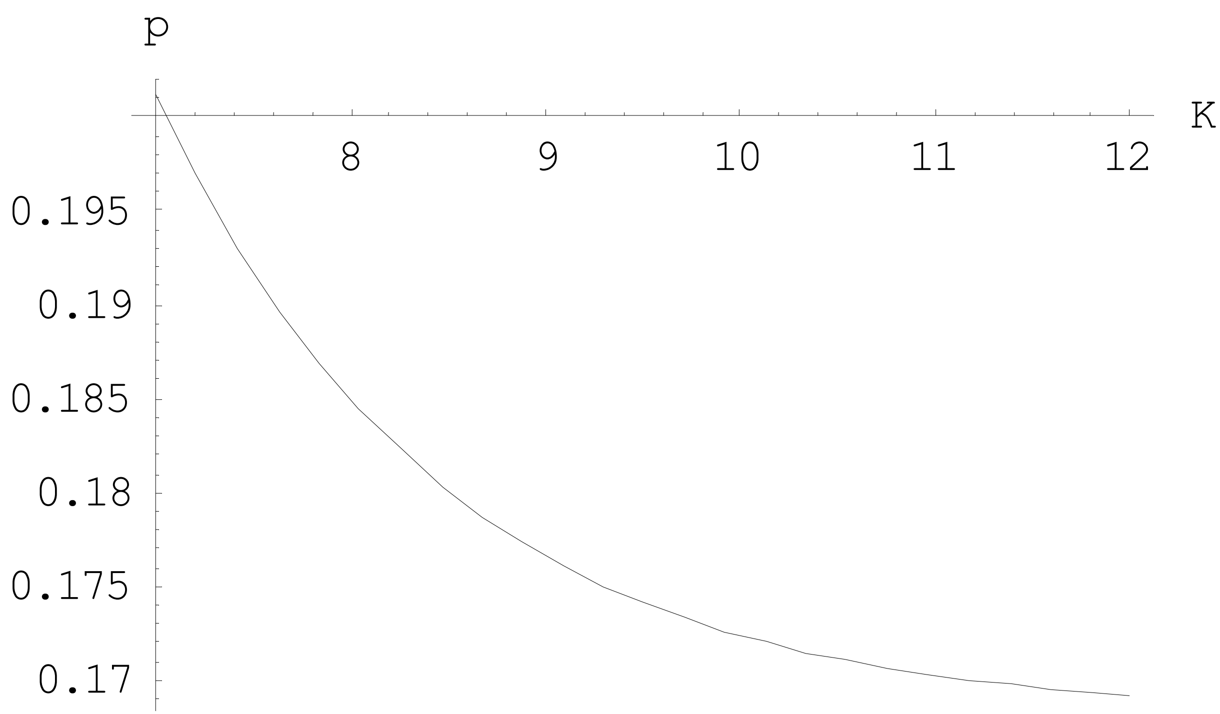

Figure 1: Example 2, p versus K

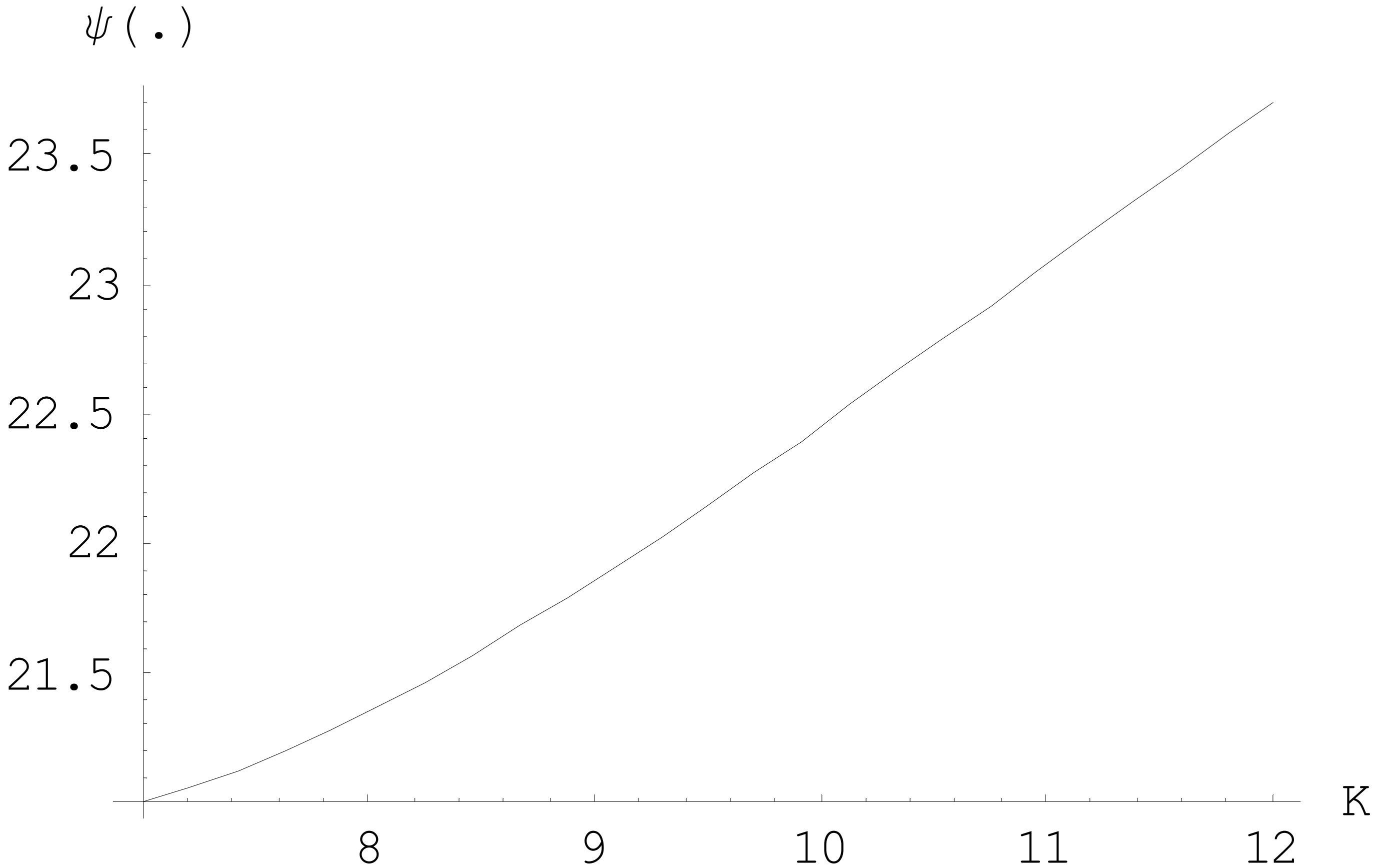

Figure 2: Example 2, $\psi(.)$ versus K

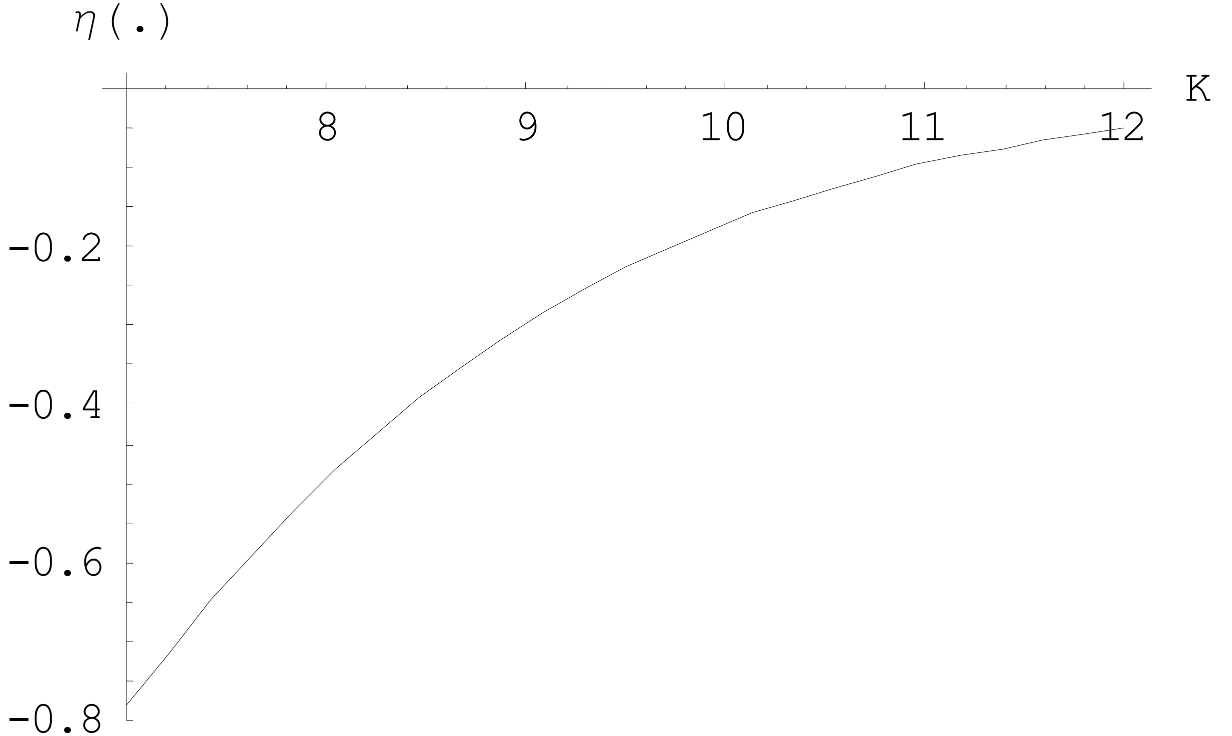

Figure 3: Example 2, $\eta(.)$ versus K

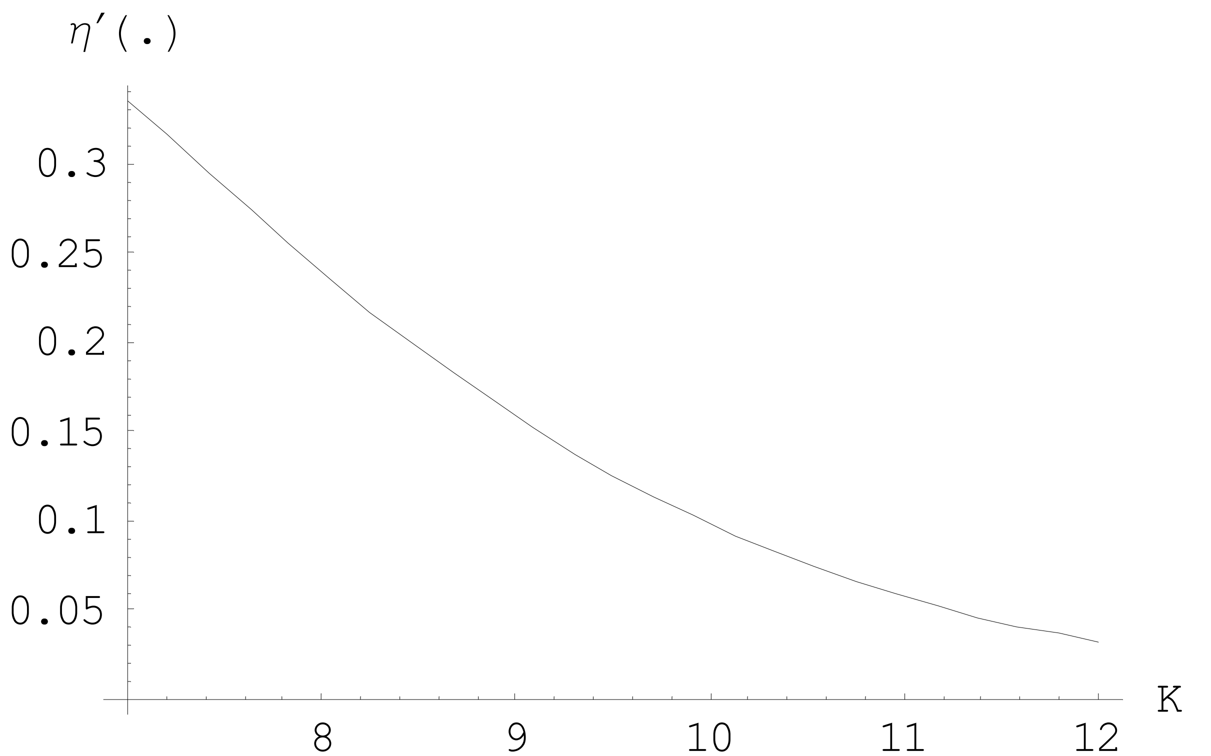

Figure 4: Example 2, $\eta'(.)$ versus K

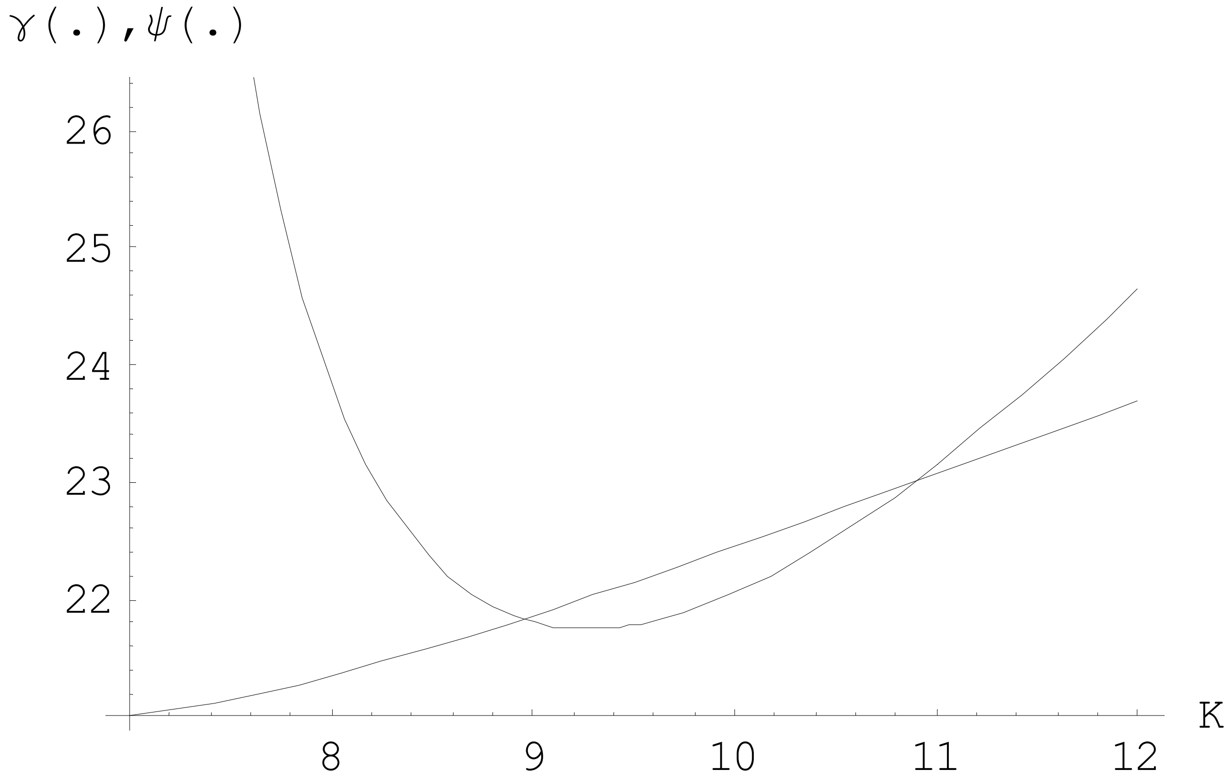

Figure 5: Example 2, Steady State Equilibria

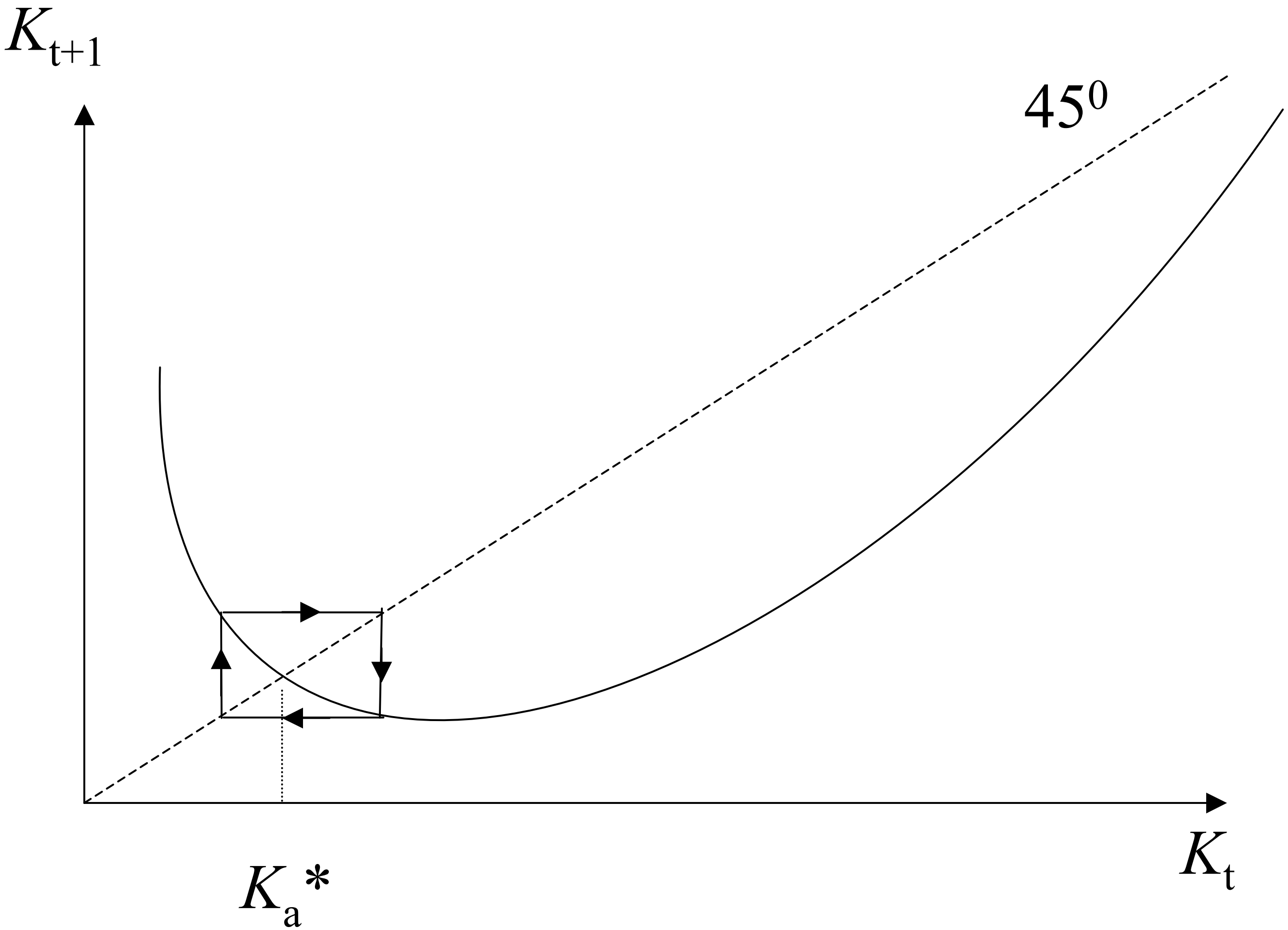

Figure 6: Example 2, Two-period Cycles

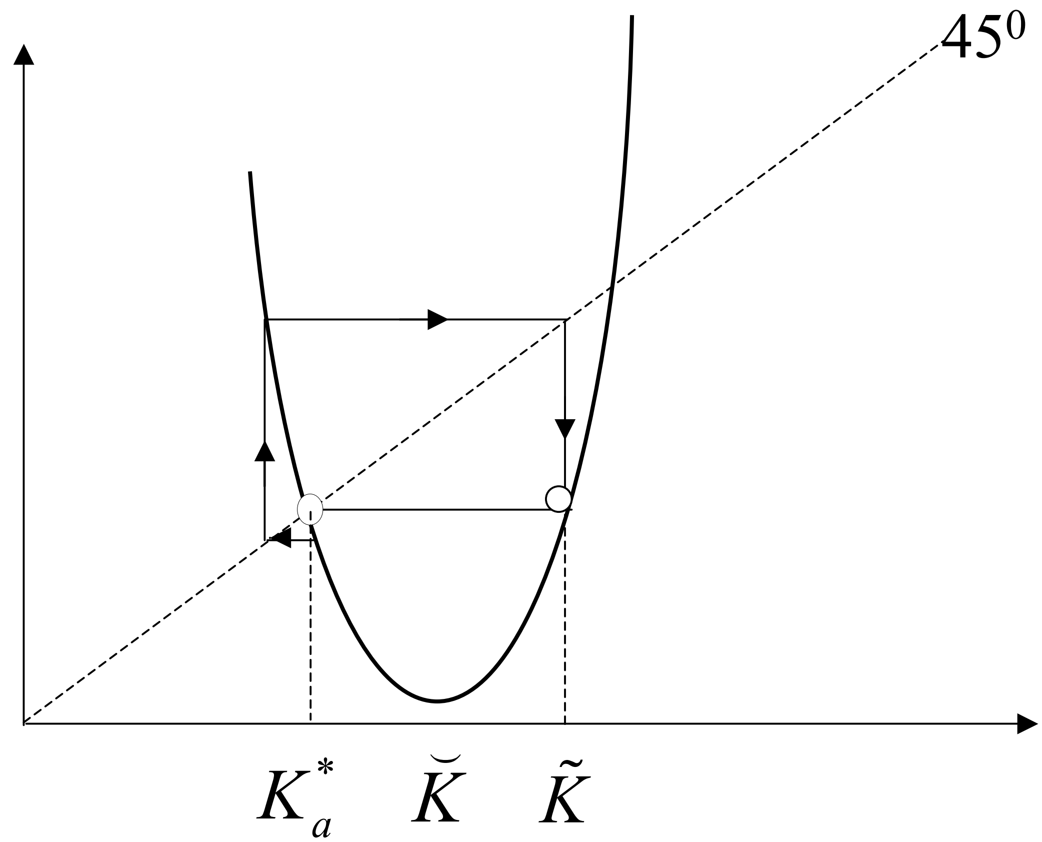

Figure 7: A snap-back repellor

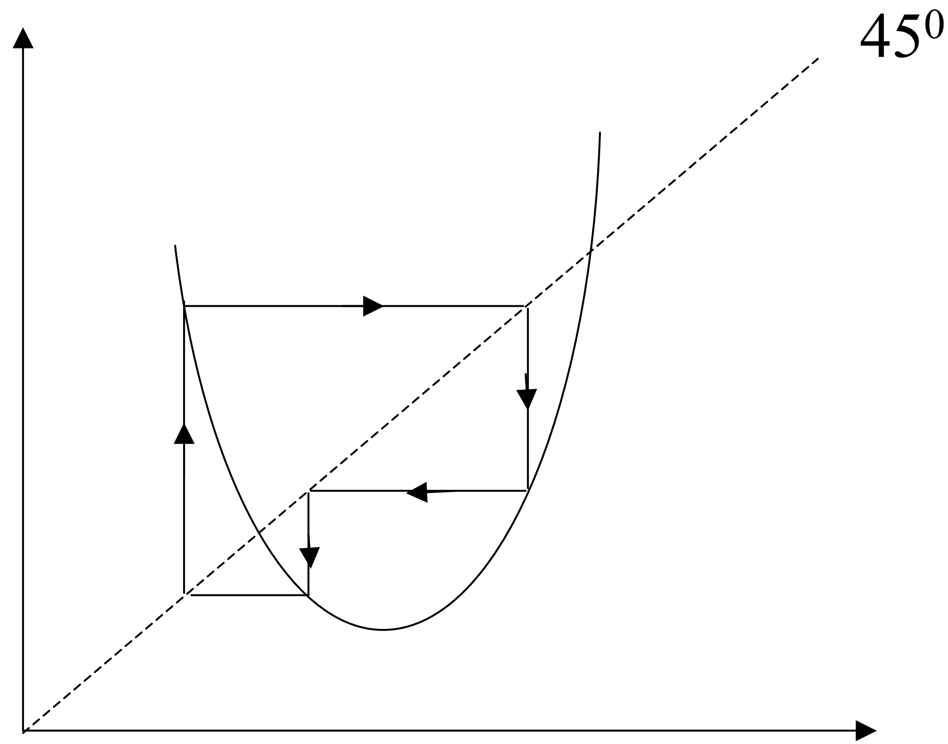

Figure 8: A three-period cycle