Unemployment and Product Market Competition in a Cournot Model with Efficiency Wage

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ABSTRACT

This paper analyzes the impact of product market competition on unemployment, wage and welfare in a model where unemployment is caused by efficiency wage considerations and oligopolistic firms compete in quantity. It is shown that while more intensive competition in product market increases output and reduces price, it does not necessarily lead to a lower unemployment rate or a higher wage for workers. Consequently, the relationship between the intensity of competition and the level of employment (respectively, wage, welfare) is not monotonic, and, in some instances, has an inverted-U shape.

Keywords: Cournot competition, unemployment, efficiency wage

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1. Introduction

How does the intensity of product market competition affect unemployment? The answer, according to most of the recent theoretical analyses such as Nickell (1999), Gerbach (2000), Blanchard and Giavazzi (2003), Ebell and Haefke (2003), and Spector (2004), is that an increase in competition intensity will reduce unemployment. An important mechanism through which increased competition affects unemployment is the “output-expansion effect.” That is, more intense competition in the product market reduces the price and expands output, which in turn increases the amount of labor employed in production.

There are, however, exceptions to this majority view on the relationship between product market competition and unemployment. They are Koskela and Stenbacka (2005) and Amable and Gatti (2000 and 2004). Koskela and Stenbacka (2005) show that reduced distortions in the product market do not necessarily improve the performance of the labor market. Specifically, intensified product market competition will hurt employment if the product market imperfection is “sufficiently strong” and the relative bargaining power of the trade union is not “high enough”. Of particular relevance to the present paper, Amable and Gatti (2000 and 2004) examine models of monopolistic competition with efficiency-wage unemployment, where both the hiring rate and job separation rate are endogenously determined. They show that an increase in product market competition may generate employment losses because it raises the job turnover rate thus causing the efficiency wage schedule to shift upward.
In this paper we contribute to the debate over the impact of product market competition on unemployment by studying a model where oligopolistic firms compete in quantity and unemployment is caused by efficiency wage considerations. Our model has two novel features. First, we abandon the monopolistic competition framework that has been used in all the existing analyses cited above, and instead model the structure of product market as a standard Cournot oligopoly. As a framework of imperfect competition, Cournot oligopoly is much more widely accepted among economists than monopolistic competition. Furthermore, this approach enables us to construct a tractable model using general demand and production functions, as opposed to the specific functional forms such as the CES utility function and Cobb-Douglas (or linear) production function commonly used in the existing models.

The second novel feature of our model is that we modify the classic Shapiro and Stiglitz (1984) efficiency wage model by assuming that the probability of detecting shirking depends on a firm’s investment in monitoring its workers. Accordingly, our modeling of efficiency wage unemployment differs from those of Amable and Gatti (2000 and 2004) in two aspects. On the one hand, we maintain Shapiro and Stiglitz’s original assumption of an exogenous job separation rate. On the other hand, we endogenize the probability of catching a shirker by making it a function of a firm’s investment in monitoring.

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1 D’Aspremont et al. (1989) study unemployment in a Cournot oligopoly model. In their model, unemployment is caused by exogenously fixed wages. In contrast, wage and unemployment in our model are determined endogenously by efficiency wage considerations.
These two features of our model are instrumental in generating the findings of our analysis. Specifically, the use of a general production function allows us to qualify the output-expansion effect of increased competition. We show that while more intense competition in product market, measured by an increase in the number of firms, reduces price and expands output, the expansion of output itself is not sufficient to guarantee higher employment unless the production technology exhibits constant returns to scale in labor. With decreasing returns to scale in labor, on the other hand, an increase in competition intensity can raise output level without augmenting the employment. Consequently, the impact of increased product market competition on unemployment is not monotonic.²

As a result of the second feature of our model, we uncover a channel for product market competition to affect unemployment that, to our knowledge, has not been identified in the literature. In our model, increased product market competition reduces the marginal benefit of monitoring and, accordingly, a firm’s incentive to invest in monitoring. This pushes up the efficiency wage schedule, which tends to reduce employment. Offsetting this tendency is the expansion of output that, under certain conditions, boosts employment.

On efficiency wage rate the simple structure of our model allows us to identify clearly three effects of product market competition. The first one, the “competition effect,” is that an increase in the number of firms bids up the efficiency wage. The second effect,

² Spector (2004) also considers the case of decreasing returns to scale in labor. In his model, increased product market competition has an ambiguous effect on wage, but has an unambiguously positive effect on employment.
the “firm size effect,” is that having more firms in the market causes each firm to produce less and hire fewer workers, which reduces the efficiency wage. Finally, the third effect, the “monitoring effect,” arises because a firm responds to intensified competition by reducing the level of monitoring. This last effect raises the efficiency wage. Hence, a sufficient condition for wage rate to rise with competition intensity is that the competition effect dominates the firm size effect (i.e., if aggregate employment rises).

The presence of monitoring costs also adds a wrinkle to the conventional wisdom that increased product market competition enhances economic efficiency. In our model, the conventional wisdom holds as long as the total monitoring costs incurred by all firms do not increase with the number of firms. Otherwise, the impact of increased product market competition on welfare is ambiguous. Numerical simulations of our model indicate a possible inverted-U relationship between competition intensity and welfare. In fact, an inverted-U is also found in our numerical simulations for the relationship between competition and employment (and respectively, wage).

This paper is organized as follows. We set up the model in section 2, and characterize the equilibrium in section 3. The impact of product market competition on the equilibrium is studied in section 4, and the results from the numerical simulations are presented in section 5. Conclusions are in section 6.
2. The Model

2.1 Consumers/Workers and the No-Shirking Constraint

Unemployment in this model is caused by efficiency wage considerations. Because workers have a tendency to shirk and firms detect shirking only imperfectly, the latter have to keep the wage above the full-employment level in order to maintain effort. In Shapiro and Stiglitz (1984), this efficiency wage is determined by the no-shirking constraint, which we now incorporate into a general equilibrium model of product market competition.

Consider an economy with two consumption goods, X and Y, and N identical consumers/workers. The utility function of each consumer takes the form \( u(x) + y - \theta e \), where \( x \) and \( y \) are the consumer’s consumption levels of goods X and Y, respectively, and \( e \) is the disutility of efforts on the job for an employed worker. Binary variable \( \theta \) indicates whether an employed worker provides efforts on the job, with \( \theta = 1 \) for the case where the worker does, and \( \theta = 0 \) otherwise. By definition, \( \theta = 0 \) when the consumer is unemployed. Function \( u \) satisfies the standard assumptions that \( u' > 0 \) and \( u'' < 0 \).

The economy is endowed with \( \bar{Y} \) units of good Y, while good X is produced using labor. Good Y is the numeraire good and its price is normalized to unity. Let \( p \) denote the price of good X, and \( w \) the wage of a worker.

Each consumer owns \( 1/N \) of the endowment \( \bar{Y} \) and \( 1/N \) of all the firms in the economy. Let \( \Pi \) denote the total profits of all firms in the economy. As a shareholder, the consumer receives a dividend of \( \Pi/N \). Furthermore, the consumer receives a wage \( w \) when he is
employed, and an unemployment insurance benefit $\bar{w}$ when he is unemployed. The unemployment insurance benefit is financed by a head tax on every consumer in the economy, denoted by $\tau$. Let $\omega$ denote the income that the consumer receives from his role as a worker, i.e., $\omega = w$ or $\bar{w}$. Then the consumer’s budget constraint can be written as

$$px + y = \omega - \tau + \Pi / N + \bar{Y} / N.$$  \hspace{1cm} (1)

From the first-order condition of the consumer’s utility-maximization problem, we obtain his inverse demand function for good $X$, $p = u'(x)$. Since the total demand by $N$ consumers is $X = Nx$, the inverse market demand function can be expressed as $p = P(X)$, where $P(X) \equiv u'(X/N)$. The assumption $u'' < 0$ implies that $P' < 0$. Let $x^*(p)$ be the solution to the consumer’s utility-maximization problem. Then we can write the consumer’s indirect utility function as:

$$[u(x^*(p)) - px^*(p) - \tau + \Pi / N + \bar{Y} / N] + \omega - \theta e.$$  \hspace{1cm} (2)

Note that in (2) the consumer’s shirking decision directly affects $\omega - \theta e$ but not the other terms. Hence, the indirect utility function (2) is qualitatively the same as the utility function used by Shapiro and Stiglitz (1984 p435). Using the same procedure as Shapiro and Stiglitz (1984), we derive the no-shirking wage constraint facing an individual firm $i$:

$$w_i = \frac{(w_{-i} - e)a + \bar{w}(b + r)}{(a + b + r)}\left[1 + \frac{b + r}{q}\right]e$$  \hspace{1cm} (3)

where $w_{-i}$ denotes the wage rate offered by other firms; $a$ is the job acquisition rate; $b$ is the separation rate; $r$ is the intertemporal discount rate; and $q$ is the probability that a shirking worker is detected.
2.2 Firms

On the production side are \( n \) identical firms that produce good \( X \), using a technology represented by

\[
x_i = F(\theta l_i), \quad \text{with} \quad F(0) = 0, F'(\cdot) > 0 \quad \text{and} \quad F''(\cdot) \leq 0,
\]

where \( l_i \) is the number of workers employed by firm \( i \) \((i = 1, 2, \ldots, n)\). Given that the wage offered by the firm satisfies the no-shirking constraint (3), \( \theta = 1 \). With a slight abuse of notation, we will use \( X \) to denote also the total output of good \( X \), \( i.e. \) \( X \equiv \sum_{i=1}^{n} x_i \).

We assume that firms are Cournot competitors, \( i.e. \), they compete by choosing quantity \( x_i \). Recall that the inverse demand function for good \( X \) is given by \( p = P(X) \). We make the standard assumption for the Cournot model that \( P'(X) + XP''(X) \leq 0 \) for all \( X \geq 0 \).\(^3\)

A well-known implication of this assumption is that each firm’s marginal revenue is decreasing in its own output as well as in its rivals’ outputs.

One novelty of this model is that we endogenize the firm’s probability of detecting shirking. Specifically, we suppose that \( q \) is a function of the investment made by the firm in monitoring the workers. Let \( m_i \) denote the level of monitoring at firm \( i \). We assume that \( q(m_i) \) satisfies \( q'(m_i) > 0 \) and \( q''(m_i) < 0 \); in other words, \( q \) is an increasing and concave function of \( m_i \). On the other hand, monitoring is costly to the firm. The costs of monitoring are represented by function \( H(m_i) \) with \( H'(m_i) > 0 \) and \( H''(m_i) > 0 \).

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\(^3\) In terms of the consumer’s utility function, this assumption is equivalent to \( u''(x) + xu'''(x) \leq 0 \).
Firm $i$’s profit can then be expressed as $\pi_i = P(X)x_i - w_i l_i - H(m_i)$. It maximizes its profit by choosing output $x_i$ and monitoring $m_i$, taking as given the no-shirking wage constraint (3) as well as the levels of output and wage of all other firms. Since labor is the only input of production, output $x_i$ is determined by $l_i$. Hence, using (3) and (4) we can write firm $i$’s optimization problem as

$$\max_{(m_i, l_i)} \pi_i = \left\{ P(F(l_i) + X_{-i})F(l_i) - \left[ \frac{(w_{-i} - e)a + \bar{w}(b + r)}{a + b + r} + \left( 1 + \frac{b + r}{q(m_i)} \right) e \right] l_i - H(m_i) \right\}$$

(5)

where $X_{-i} \equiv \sum_{j=1}^{n} x_j - x_i$ is the total output of firm $i$’s rivals. The first-order conditions for firm $i$’s profit-maximizing problem (5) are:

$$\frac{e \cdot q'(m_i)}{[q(m_i)]^2} (b + r) l_i = H'(m_i)$$

(6)

$$[P'(X)F(l_i) + P(X)]F'(l_i) = \left[ \frac{(w_{-i} - e)a + \bar{w}(b + r)}{a + b + r} + \left( 1 + \frac{b + r}{q(m_i)} \right) e \right]$$

(7)

Equation (6) implies that each firm sets its level of monitoring such that the marginal benefit of monitoring equals the marginal cost of monitoring; while equation (7) indicates that every firm will choose the number of workers such that the marginal revenue product ($MRP$) equals the no-shirking wage. Let $(m_i^*, l_i^*)$ denote the solution to the first-order conditions (6) and (7).
Regarding the second-order conditions of (5), it is shown in Appendix that
\[
\frac{\partial^2 \pi_i}{\partial m_i^2} < 0, \quad \frac{\partial^2 \pi_i}{\partial m_i \partial l_i} = \frac{\partial^2 \pi_i}{\partial l_i \partial m_i} > 0 \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial l_i^2} < 0.
\]
To ensure that all second-order conditions are satisfied, we assume
\[
\left[ \frac{\partial^2 \pi_i}{\partial l_i^2}, \frac{\partial^2 \pi_i}{\partial m_i^2} \right]_{(m_i^*, l_i^*)} > \left[ \frac{\partial^2 \pi_i}{\partial l_i \partial m_i} \right]_{(m_i^*, l_i^*)}^2.
\]

3. Equilibrium

Since all consumers and firms are identical in this economy, we focus on a symmetric equilibrium where all firms produce the same quantity and all consumers consume the same amounts of X and Y. Hence, we set \( m_i^* = m, \ l_i^* = l \) and \( w_i^* = w \) for all \( i \). Then from the no-shirking constraint (3) we obtain
\[
w = \bar{w} + e + \frac{e}{q(m)} (a + b + r) \tag{9}
\]

In the steady state of the labor market, the flow into the unemployment pool per unit time is equal to the flow out of the unemployment pool per unit time. That is, \( bL = a(N - L) \), where \( L = nl \) is the aggregate employment. We use this condition to rewrite (9) as
\[
w = \bar{w} + e + \frac{e}{q(m)} \left( \frac{bN}{N - nl} + r \right) \tag{10}
\]
Condition (10) is the efficiency wage schedule.
Substituting (10) into (7) and setting \(m_i = m\) and \(l_i = l\) in (6) – (7), we obtain the following two equilibrium conditions:

\[
\frac{eq'(m)}{[q(m)]^2}(b + r)l = H'(m) \tag{11}
\]

\[
[P'\left(nF(l)\right)F(l) + P\left(nF(l)\right)]F'(l) = \left[\bar{w} + e + \frac{e}{q(m)} \left(\frac{bN}{N-nl} + r\right)\right] \tag{12}
\]

Note that (12) implies \(pF' > w > e\); that is, the value of marginal product of labor exceeds the efficiency wage, which in turn exceeds the utility cost of efforts. These inequalities are the result of imperfections in the product market and the labor market. Imperfect competition in the product market raises price above marginal cost, and imperfect information in the labor market causes the wage to exceed the opportunity cost of efforts.

Equation system (11) – (12) determines the equilibrium levels of monitoring and employment by each firm, \((m^*, l^*)\). Substituting them into (4) and (10) we obtain a firm’s output level \(x^*\) and wage rate \(w^*\). From there we find the equilibrium quantity \(X^*\) and price \(p^*\) in the market for good X. By Walras’ law, the market for Y is also in equilibrium at price \(p^*\).

The equilibrium is illustrated in Figure 1, where the curve \(l = A(m)\) represents (11) and the curve \(l = B(m)\) represents (12). It is demonstrated in the appendix that that both curves are upward-sloping, but \(A(m)\) is steeper than \(B(m)\) at the intersection. This implies that the equilibrium is unique.

Intuitively, (11) yields a positive relationship between \(l\) and \(m\) because the marginal benefit of monitoring increases with \(l\) and consequently, a larger \(l\) encourages the firm to
increase the level of monitoring. On the other hand, (12) also implies a positive relationship between these two variables because a higher level of monitoring reduces the no-shirking wage and thus induces firms to hire more workers.

4. Effects of Product Market Competition

In this section we examine the impact of product market competition on equilibrium employment level, the wage rate, output level and price. In our model the intensity of product market competition is measured by the number of firms, \( n \). Accordingly, the impact of competition intensity is determined by conducting comparative statics with respect to \( n \).

Note that \( n \) appears in (12) but not in (11). In fact, an increase in \( n \) causes the curve \( l = B(m) \) to shift downward. As shown in Figure 2, this downward shift moves the equilibrium from point \( E \) to point \( \bar{E} \). Hence, we have

**Proposition 1.** Increased product market competition reduces both the number of workers employed and the level of monitoring at each firm.

Intuitively, an increase in the number of firms reduces the marginal revenue product of labor at each firm (the left-hand side of (12)) and raises the no-shirking wage (the right-hand side of (12)). Both cause a firm to reduce the number of workers it employs. Fewer workers at a firm decreases the marginal benefit of monitoring, inducing the firm to

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4 An increase in the number of firms could be the result of product market deregulation that lowers the barriers to entry into the market for good \( X \). We can easily extend our model to allow free entry with an exogenously fixed entry cost. It can be shown that a reduction in the entry cost in this extended model has qualitatively the same effects as an increase in the number of firms in the present model.

5 The formal proofs of this and all subsequent propositions are relegated to Appendix.
lower its level of monitoring. The fall in the level of monitoring shifts up the efficiency wage schedule, which causes the employment level at each firm to fall even further.

Since labor is the only factor of production, Proposition 1 implies that the output level of each firm decreases with the number of firms. The aggregate output level, however, is higher because the reduction in output by each firm is offset by the larger number of firms. Price falls as a consequence. In other words,

**Proposition 2.** Increased product market competition raises the total output level and lowers the price of good X in equilibrium.

An important mechanism through which increased competition affects employment is the output-expansion effect. Intuition suggests that the output expansion brought about by more intense competition should lead to an expansion of employment as well. In our model, this intuition is correct as long as the production technology exhibits constant returns to scale in labor (i.e., $F'' = 0$), in which case there is a one-to-one relationship between total output and aggregate employment. In the case of diminishing returns to labor (i.e., $F'' < 0$), however, a mere reallocation of a fixed number of workers evenly among a larger number of firms will increase total output. If such an increase in total output is sufficiently large to meet the expansion in demand arising from the lower price induced by intensified competition, aggregate employment will not expand in response to a larger number of firms.

To gain further understanding about the condition under which intensified competition would reduce aggregate employment, consider (12) which states that the marginal revenue product of labor is equal to efficiency wage. Note that the efficiency wage (the
right-hand side of (12)) increases with aggregate employment but decreases with the level of monitoring. Since an increase in the number of firms reduces the level of monitoring, a falling wage accompanying a larger $n$ would imply a contraction in aggregate employment. For that to happen, the left-hand side of (12), the marginal revenue product of labor, would have to fall as well. On the marginal revenue product an increase in the number of firms has two effects, a direct effect and an indirect effect. The direct effect is that a larger $n$ directly reduces the value of the left-hand side of (12). The indirect effect is through the reduction in the number of workers hired by each firm, and this indirect effect raises value of the left-hand side of (12). The marginal revenue product, and hence the aggregate employment, will decrease if the direct effect dominates the indirect effect.

To formalize this condition, let $MRP^*$ denote the left-hand side of (12) and define the following two elasticities that are related to the direct and indirect effects of a larger $n$: $\varepsilon_n = -(\partial MRP^* / \partial n)(n / MRP^*)$ and $\varepsilon_i = -(\partial MRP^* / \partial l)(l^* / MRP^*)$. Then we have

**Proposition 3.** Increased product market competition may either raise or reduce aggregate employment. Aggregate employment rises with competition intensity if $F'' = 0$. It falls with competition intensity if $\varepsilon_n > \varepsilon_i$.

To ascertain the effects of product market competition on the wage rate, consider the efficiency wage schedule (10). An increase in the number of firms has three effects on the efficiency wage. The first one, which we call the “competition effect,” is that a larger $n$ directly raises the efficiency wage. The second effect, which will be termed the “firm size effect,” is that intensified competition causes each firm to hire fewer workers, which
reduces the efficiency wage. Finally, the third effect, the “monitoring effect,” arises because a firm responds to intensified competition by reducing the level of monitoring. This last effect raises the efficiency wage. Hence, a sufficient condition for wage rate to rise with competition intensity is that the competition effect dominates the firm size effect (i.e., if aggregate employment, \( n_l \), rises).

**Proposition 4.** Increased product market competition may either raise or reduce wage rate. It raises wage rate as long as it also expands aggregate employment.

Since consumers own all firms and endowments in this economy, welfare can be measured by the sum of utilities of all consumers. That is,

\[
W = L[u(x) + w + n\pi / N + \bar{Y} / N - \tau - e] + (N - L)[u(x) + \bar{w} + n\pi / N + \bar{Y} / N - \tau]
\]  

Using the consumer’s optimization condition, the definition of the firm’s profit, and the government’s budget constraint, \((N - L)\bar{w} = N\tau\), we can rewrite \(W\), evaluated at the equilibrium values, as:

\[
W = \int_0^{X^*} P(X)dX + \bar{Y} - eL^* - nH(m^*)
\]

Intuitively, there are two types of market imperfections in this model, one in the labor market and the other in the product market. In the labor market, wage rate exceeds the opportunity cost of efforts due to efficiency wage. Thus, efficiency in the labor market will be improved if aggregate employment increases. In the product market, price exceeds marginal cost due to imperfect competition. In the absence of any fixed cost, an increase in the number of firms that lowers price and increases total quantity would
improve efficiency in the product market. However, in this model the monitoring costs $H(m)$ do not vary with the output level of each firm. Then the improved efficiency from a smaller price-cost margin has to be balanced against the possible increase in total monitoring costs $nH(m)$ resulted from a larger $n$. Hence we have the following result about the welfare effect of increased product market competition.

**Proposition 5.** Increased product market competition may either increase or decrease welfare. It increases welfare as long as it reduces the total costs of monitoring $nH(m)$.

To summarize, the analysis in this section shows that while more intense product market competition increases the total output and reduces the price of good X, it has an ambiguous impact on aggregate employment, wage rate and welfare. On the latter, we find sufficient conditions under which competition has a positive impact on these variables. In the next section, we will present numerical examples that demonstrate the possibility that increased product market competition can have the opposite impact on aggregate employment, wage rate, or welfare.

5. **Simulations**

In this section, we conduct numerical simulations of our model with specific functional forms. The objectives of these simulations are to provide concrete examples for Propositions 3 - 5 where product market competition has an ambiguous impact, and more significantly, to demonstrate the existence of an inverted-U relationship between competition intensity and each of aggregate employment, welfare and wage rate.
Suppose the utility function over the consumption of good X takes the form
\[ u(x) = A - B x^{\tau+1} / (\tau + 1) \]
and the production function takes the form \( F(l) = s l^\rho \), where \( A, B, \tau, s, \) and \( \rho \) are positive constants and \( \rho \leq 1 \). The monitoring technology is represented by \( q(m) = \delta m / (m + \theta) \), with \( 0 < \delta \leq 1 \) and \( \theta > 0 \). The monitoring cost is \( H(m) = m^\gamma \) with \( \gamma > 1 \).

By choosing specific values for the parameters in these functions and other parameters in this model, we conducted numerical simulations to confirm the general results derived in section 4. Here we present two specific examples that are derived using the parameter values contained in Table 1.

**TABLE 1. Parameter Values**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>B</th>
<th>r</th>
<th>( \bar{w} )</th>
<th>N</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \theta )</th>
<th>A</th>
<th>B</th>
<th>( \tau )</th>
<th>( \rho )</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>0.05</td>
<td>0.05</td>
<td>0.5</td>
<td>500</td>
<td>2</td>
<td>0.8</td>
<td>5</td>
<td>275</td>
<td>0.275</td>
<td>1.5</td>
<td>0.95</td>
<td>0.35</td>
</tr>
<tr>
<td>Example 2</td>
<td>0.05</td>
<td>0.05</td>
<td>0.5</td>
<td>500</td>
<td>2</td>
<td>1</td>
<td>2.5</td>
<td>100</td>
<td>0.4</td>
<td>1.2</td>
<td>0.65</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Example 1 is illustrated in Figures 3, 4 and 5. From Figures 3 and 4 we see that the relationships between competition and employment, and between competition and welfare, are not monotonic. To be more specific, these relationships have the shape of an inverted-U. That is, employment (respectively, welfare) rises with competition intensity for \( n \) below a certain threshold, but the opposite is true if \( n \) exceeds the threshold. However, the relationship between competition and wage is a monotonic one in this example (see Figure 5). On the other hand, example 2 illustrated in Figure 6 shows that the relationship between these two variables can also display an inverted-U shape.
6. Conclusions

In this paper, we have analyzed the employment and efficiency consequences of product market competition using a model where oligopolistic firms compete in quantity and unemployment is caused by efficiency wage considerations. We have shown that more intense competition in the product market generates a larger total output and thus a lower price. However, unless production technology exhibits constant-returns-to-scale, intensified competition does not necessarily create more jobs. Nor does it necessarily raise wage or improve welfare. Numerical simulations of our model indicate that the relationship between the intensity of product market competition and each of these variables can, in some instances, have an inverted-U shape.
REFERENCES


Appendix

1. Slopes of the A(m) and B(m) Curves

To obtain the slope of \( l = A(m) \), we totally differentiate (11) with respect to \( m \) and \( l \),

\[
\frac{dl}{dm} = \frac{2el\left[q\right]^2\left(\frac{bN}{N-nl} + r\right) - elq''\left(\frac{bN}{N-nl} + r\right) + H''}{(b+r)eq'/q^2}
\]

(A1)

By assumptions \( q' > 0, q'' < 0 \) and \( H'' > 0 \), it is clear that both the numerator and the denominator of (A1) are positive. Thus \( dl/dm > 0 \), that is, the \( A(m) \) curve is upward-sloping.

Similarly, totally differentiating (12) with respect to \( m \) and \( l \) and then rearranging, we obtain the slope of \( l = B(m) \),

\[
dl/dm = -\left\{eq'q^{-2}[bN(N-nl)^{-1} + r]\right\} / \left\{\left[P'\right] \left[(n+1)P' + nFP''\right] + \left[P + FP'\right]F'' - bnq^{-2}(N-nl)^{-2}\right\}
\]

(A2)

Note that the assumptions \( P'(X) < 0 \) and \( P'(X) + XP''(X) \leq 0 \) imply that each firm’s marginal revenue is decreasing in the total output of other firms, that is, \( P'(X_{-i} + x_i) + x_iP''(X_{-i} + x_i) < 0 \), which in turn implies that the denominator of (A2) is negative. Since the numerator is also negative, \( dl/dm > 0 \). That is, the \( B(m) \) curve is also upward-sloping.

2. The Uniqueness of Equilibrium

First, note that the second-order derivatives of a firm’s profit function in (5) are:
\[
\frac{\partial^2 \pi_i}{\partial m_i^2} \bigg|_{(m_i^*, l_i^*)} = e \left( \frac{2 [q'(m_i^*)]^2}{q(m_i^*)} + \frac{q''(m_i^*)}{[q(m_i^*)]^2} \right) \left( b + r \right) l_i^* - H''(m_i^*) < 0 \tag{A3}
\]

\[
\frac{\partial^2 \pi_i}{\partial m_i \partial l_i} \bigg|_{(m_i^*, l_i^*)} = \frac{e q'(m_i^*)}{q(m_i^*)} \left( b + r \right) > 0 \tag{A4}
\]

\[
\frac{\partial^2 \pi_i}{\partial l_i^2} \bigg|_{(m_i^*, l_i^*)} = \left[ P'' \left( F(l_i^*) + Y_{-i} \right) F(l_i^*) + 2 P' \left( F(l_i^*) + Y_{-i} \right) \right] \left[ F'(l_i^*) \right]^2
\]

\[
+ \left[ P \left( F(l_i^*) + Y_{-i} \right) + P' \left( F(l_i^*) + Y_{-i} \right) F(l_i^*) \right] F''(l_i^*) < 0 \tag{A5}
\]

Let \( \Phi = 0 \) and \( \Omega = 0 \) represent the equilibrium conditions (11) and (12), and let \( \Phi_j \) and \( \Omega_j \) \((j = m, l, n)\) denote the partial derivative of \( \Phi \) and \( \Omega \) with respect to variable \( j \).

Differentiating \( \Phi \) and \( \Omega \) and using (A3) – (A5), we obtain

\[
\Phi_i = \frac{\partial^2 \pi_i}{\partial l_i \partial m_i} \bigg|_{(m_i^*, l_i^*)} = \frac{e q'(m_i^*)}{q(m_i^*)} \left( b + r \right) > 0 \tag{A6}
\]

\[
\Phi_m = \frac{\partial^2 \pi_i}{\partial m_i^2} \bigg|_{(m_i^*, l_i^*)} = e \left( \frac{2 [q'(m_i^*)]^2}{q(m_i^*)} + \frac{q''(m_i^*)}{[q(m_i^*)]^2} \right) \left( b + r \right) l_i^* - H''(m_i^*) < 0 \tag{A7}
\]

\[
\Omega_j = \frac{\partial^2 \pi_i}{\partial l_i^j} \bigg|_{(m_i^*, l_i^*)} + \frac{enbN}{q(m_i^*) (N - nl^*)} + (n - 1) \left[ P' \right]^2 \frac{P' \left( nF(l_i^*) \right) + F(l_i^*) P'' \left( nF(l_i^*) \right)}{(nF(l_i^*))^2} < 0 \tag{A8}
\]

\[
\Omega_m = \frac{\partial^2 \pi_i}{\partial l_i \partial m_i} \bigg|_{(m_i^*, l_i^*)} + \frac{eq'(m_i^*)}{q(m_i^*)} \frac{nbl}{(N - nl^*)} > 0 \tag{A9}
\]

\[
\Phi_n = 0 \tag{A10}
\]

\[
\Omega_n = F(l_i^*) F'(l_i^*) \left[ P' \left( nF(l_i^*) \right) + F(l_i^*) P'' \left( nF(l_i^*) \right) \right] - \frac{enbNl^*}{q(m_i^*) (N - nl^*)} < 0 \tag{A11}
\]
The sign of $\Omega_n$ is negative because $P'(nF(l')) + F(l')P''(nF(l')) < 0$. The slopes of the $A(m)$ and $B(m)$ curves at their intersection can be written as $A'(m^*) = -\Phi_m / \Phi_i$ and $B'(m^*) = -\Omega_m / \Omega_i$. Note that

\[
\left[ \Omega - \frac{\partial^2 \pi_i}{\partial l^2} \right]_{\Omega_m} > \left[ \frac{-e}{q(m^*)} \frac{nbN}{(N-nl^*)^2} \right] \left\{ \frac{2elq(m^*)}{[q(m^*)]^3} (b + r) \right\} > \left\{ \frac{eq(m^*)}{[q(m^*)]^2} \frac{nbl(b + r)}{N - nl^*} \right\} \tag{A12}
\]

and

\[
\Phi_i \left[ \Omega_m - \frac{\partial^2 \pi_i}{\partial l \partial m_l} \right]_{\Omega_m} \left\{ \frac{eq(m^*)}{[q(m^*)]^2} \frac{nbl(b + r)}{N - nl^*} \right\} \tag{A13}
\]

Condition (8), along with (A12) and (A13), implies

\[
\left[ \frac{\partial^2 \pi_i}{\partial l^2} \frac{\partial^2 \pi_i}{\partial m_l^2} \right]_{\Omega_m} > \left[ \Omega - \frac{\partial^2 \pi_i}{\partial l^2} \right]_{\Omega_m} > \left[ \frac{eq(m^*)}{[q(m^*)]^2} \frac{nbl(b + r)}{N - nl^*} \right] \tag{A14}
\]

Using (A6) – (A9) we obtain

\[
\Omega_i \Phi_m = \left[ \frac{\partial^2 \pi_i}{\partial l^2} \frac{\partial^2 \pi_i}{\partial m_l^2} \right]_{\Omega_m} + \left[ \Phi_i \left[ \Omega_m - \frac{\partial^2 \pi_i}{\partial l^2} \right]_{\Omega_m} \right] \Phi_m \tag{A15}
\]

\[
\Phi_i \Omega_m = \left[ \frac{\partial^2 \pi_i}{\partial l^2} \frac{\partial^2 \pi_i}{\partial m_l^2} \right]_{\Omega_m} + \Phi_i \left[ \Omega_m - \frac{\partial^2 \pi_i}{\partial l^2} \right]_{\Omega_m} \tag{A16}
\]

Then (A14) – (A16) imply that $\Omega_i \Phi_m > \Phi_i \Omega_m$, or equivalently, $-\Phi_m / \Phi_i > -\Omega_m / \Omega_i$. Hence, the slope of $A(m)$ is greater than that of $B(m)$ at their intersection. This implies that these curves can intersect only once. In other words, the equilibrium is unique.

Comparative statics on (11) and (12) yields,

\[
\frac{dm^*}{dn} = \frac{\Phi_i \Omega_n - \Omega_i \Phi_n}{\Omega_i \Phi_m - \Phi_i \Omega_m} = \frac{\Phi_i \Omega_n}{\Omega_i \Phi_m - \Phi_i \Omega_m} 
\]  \hspace{1cm} (A17)

\[
\frac{dl^*}{dn} = \frac{\Omega_m \Phi_n - \Phi_m \Omega_n}{\Omega_i \Phi_m - \Phi_i \Omega_m} = \frac{-\Phi_m \Omega_n}{\Omega_i \Phi_m - \Phi_i \Omega_m} 
\]  \hspace{1cm} (A18)

We have shown above that \( \Omega_i \Phi_m > \Phi_i \Omega_m \), implying that the denominators of (A17) and (A18) are positive. To determine the sign of (A17), note that \( \Phi_i > 0 \) from (A6) and \( \Omega_n < 0 \) from (A11). Therefore, we have \( dm^*/dn < 0 \). In (A18), note that \( \Omega_n < 0 \) and \( \Phi_m < 0 \) from (A7). Hence, \( dl^*/dn < 0 \).

4. Proof of Proposition 2

Since price is a decreasing function of quantity, it suffices to show that a larger \( n \) raises total output of \( X \). In the case where a larger \( n \) leads to a larger aggregate employment, total output must be higher. This is because with \( F'' \leq 0 \), total output will not fall if the same amount of labor is allocated evenly among more firms. Consequently, a larger aggregate employment can only lead to a larger total output.

Next, consider the case where a larger \( n \) leads to a fall in aggregate employment. Suppose that this increase in \( n \) has led to a reduction in total output \( X^* \). Consider equilibrium condition (12). The reduction in \( X^* \) raises the marginal revenue product of labor (the left-hand side of (12)) for a firm, and the fall in aggregate employment reduces
the efficiency wage (the right-hand side of (12)). Both imply that each firm should hire more workers and increase its output. Given that the number of firms is larger, this increase in output by each firm should only increase $X^*$, which contradicts the supposition that $X^*$ falls in response to an increase in $n$.

5. Proof of Proposition 3

Using (A18) we obtain

$$
\frac{dL^*}{dn} = I^* + n \frac{dI^*}{dn} = I^* \left[1 + \frac{n}{I^*} \frac{dI^*}{dn}\right] = I^* \left[1 - \frac{\Phi_m \Omega_n}{\Omega_m \Phi_m - \Phi_m \Omega_m} \frac{n}{I^*} \right]
$$

(A19)

The sign of (A19) is ambiguous. It is negative if

$$
n \Phi_m \Omega_n - I^* [\Omega_m \Phi_m - \Phi_m \Omega_m] = I^* \Phi_m \Omega_m + \Phi_m \left[n \Omega_n - I^* \Omega_n\right] > 0.
$$

(A20)

Since $\Phi_m \Omega_m > 0$ and $\Phi_m < 0$, a sufficient condition for (A20) to hold is

$$
\left[n \cdot \Omega_n - I^* \cdot \Omega_n\right] < 0.
$$

Using (A5), (A8) and (A11), one can verify that this inequality is equivalent to $n \frac{\partial MRP^*}{\partial n} < I^* \frac{\partial MRP^*}{\partial I^*}$. Hence, $dL^*/dn < 0$ if $\varepsilon_n > \varepsilon_I$.

In the case of $F'' = 0$, $X = nF(l) = F(nl)$ and there is a one-to-one relationship between $X$ and $nl$. Proposition 2 implies that aggregate employment must increase in response to a larger $n$.

6. Proof of Proposition 4

Differentiating the equilibrium wage equation (10) with respect to $n$, we obtain
Recalling (A17) and (A19), we conclude that the sign of (A21) is in general ambiguous.

But a sufficient condition for \( \frac{dw}{dn} > 0 \) is \( \frac{dL}{dn} > 0 \).

7. Proof of Proposition 5

Recall that the equilibrium levels of aggregate employment \( (L^*) \), total output \( (X^*) \), monitoring \( (m^*) \) and efficiency wage rate \( (w^*) \) are all functions of \( n \). Differentiate (14) with respect to \( n \),

\[
\frac{dW}{dn} = \left( \frac{dX}{dn} \right) P^* - e \left( \frac{dL}{dn} \right) - \frac{d[nH(m^*)]}{dn}
\]

(A22)

In (A22), \( \frac{dX}{dn} \) captures the effect of a lower price brought about by increased competition. But the signs of the other two terms are ambiguous. First, suppose \( \frac{dL}{dn} \leq 0 \), then \( \frac{dW}{dn} > 0 \) as long as \( \frac{d[nH(m)]}{dn} < 0 \). Second, suppose \( \frac{dL}{dn} > 0 \). Rewrite (A22) by using \( X = nF(l) \) and by adding and subtracting \( l^*F'P^* \),

\[
\frac{dW}{dn} = (F - l^*F')P^* + (P'F' - e) \left( \frac{dL}{dn} \right) - \frac{d[nH(m^*)]}{dn}
\]

(A23)

Note that \( P'F' > (P + P'F')F' = w > e \), and that \( F > l^*F' \) because \( F'' < 0 \) and \( F(0) = 0 \). These imply that the first two terms in (A23) are positive. Hence, \( \frac{dW}{dn} > 0 \) as long as \( \frac{d[nH(m)]}{dn} < 0 \).
Finally, note that
\[
\frac{d[nH(m^*)]}{dn} = H(m^*) + nH'(m^*) \frac{dn^*}{dn},
\]
which, by (A17), has an ambiguous sign.
FIGURE 1 The Uniqueness of Equilibrium

FIGURE 2 Impact of Increased Competition on $m$ and $l$
FIGURE 3 Impact of Increased Competition on Aggregate Employment

FIGURE 4 Impact of Increased Competition on Welfare
FIGURE 5 Impact of Increase Competition on Wage Rate: Example 1

FIGURE 6 Impact of Increased Competition on Wage Rate: Example 2