McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

Examiners: Profs Ngo Van Long and Licun Xue
Location: Leacock 424
Date and Time: Friday, June 1^{st}, 2012, 12:00am-3:30pm.

Instructions:

- Answer two questions from Part A and both questions in Part B.
- All questions have equal weights.
- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 6 pages, including this cover page.

Good luck!

Let $\beta \geq 0$ be the multiplier associated with the I.C.C. constraint and let $\lambda \geq 0$ be the multiplier associated with the non-negative expected profit constraint.

1. Find the necessary conditions for the planner's constrained optimization problem.

2. Show that the Lagrange multipliers are strictly positive at the solution. Give an intuitive explanation of this result.

3. Assume that the ratio π_i/p_i is increasing in i. What can be said about the properties of the optimal deductible amounts, $D_i - C_i$, $i = 1, 2, ..., n$?

Question 2

Consider a consumer who has the following quasi-linear utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$, where

$$u(q, m) = m + \phi(q)$$

where $\phi : \mathbb{R}_+ \to \mathbb{R}$ is a continuous function with the properties that $\phi'(q) > 0$, $\phi''(q) < 0$ for all $q > 0$ and

$$\lim_{q \to 0} \phi'(q) = \infty \text{ and } \lim_{q \to \infty} \phi'(q) = 0.$$

Here $q \geq 0$ denotes her consumption of bread and $m \geq 0$ denotes her consumption of a composite commodity. The price of bread is $p > 0$ and the price of the composite commodity is $p_m = 1$. The budget constraint is

$$pq + m = y$$

The consumer wants to maximize utility subject to both (i) the *non-negativity constraints* $q \geq 0$ *and* $m \geq 0$, and (ii) the budget constraint.

Define q^{**} to be the value of q that solves the equation

$$\frac{\frac{\partial u(q,m)}{\partial q}}{p} = \frac{\frac{\partial u(q,m)}{\partial m}}{1}$$

i.e. q^{**} equates the marginal utility per dollar spent on bread to the marginal utility per dollar spent on the composite good. Obviously, q^{**} depends on p, so we can write $q^{**}(p)$.

2. Compute a Walrasian equilibrium of the economy with consumption externality. Find the allocation vector in this equilibrium. Denote it by $\overline{\mathbf{x}} = (\overline{x}_1^A, \overline{x}_2^A, \overline{x}_1^B, \overline{x}_2^B)$.

3. The Walrasian equilibrium allocation is not Pareto efficient. Show that by a suitable choice of $\delta > 0$ and $\varepsilon > 0$, it is possible to improve A's utility level without reducing B's utility level, by changing the allocation from $\overline{\mathbf{x}}$ to $\widehat{\mathbf{x}}$ where

$$\left(\widehat{x}_1^A, \widehat{x}_2^A, \widehat{x}_1^B, \widehat{x}_2^B\right) = \left(\frac{1+2\delta}{2}, \frac{1-2\varepsilon}{2}, \frac{1-2\delta}{2}, \frac{1+2\varepsilon}{2}\right)$$

Question 4

An expected utility maximizing investor must decide how to allocate his initial wealth $w > 0$ between a risky asset and a safe asset. Let x be the amount he invests in the risk asset and $w - x$ the amount he invests in the safe asset. x is constrained to lie in $[0, w]$. The *gross* rate of return on the safe asset is 1 [thus, \$1 invested in the safe asset yields a gross return of \$1]. The *gross* rate of return on investment in the risky asset is $1 + H > 1$ with probability p and $1 + L < 1$ with probability $1 - p$. The individual's utility function over net wealth m is $u(m)$ with $u'(m) > 0 > u''(m)$ for all $m > 0$.

(1) Suppose that the expected net rate of return on the risky asset, $pH + (1-p)L$, is strictly positive. Prove that the individual will hold a strictly positive amount of the risky asset.

(2) Now suppose that $u(w) = \ln(w)$. Assume that the optimal amount of the risky asset x^* satisfies $w > x^* > 0$ (i.e., there is an interior solution). Show that if H increases then so does the optimal amount of the risky asset x^*.

Question 5

Dan and Eva consume cheese (c) and wine (w). Dan's utility function is $U^D(c, w) = \ln c + w$ and Eva's utility function is $U^E(c, w) = c^2 w$, where c is measured in pounds and wine is measured in liters. Dan has 38 pounds of cheese and 3 liters of wine while Eva has no cheese and 27 litters of wine.

(1) Draw the Edgeworth box for Dan and Eva. Label their initial endowment point.

(3) Does there exist a separating perfect Bayesian equilibrium? If so, fully specify such an equilibrium; if not, prove why not.

Question 2.

Consider an individual who owns a warehouse that is subject to fire damage. The warehouse is worth \$10,000 and fire can result in a damage of \$9,100. The owner of this warehouse can take precautions against fire – for simplicity, we assume that he can be "negligent", "moderately cautious", or "very cautious". The owner's choice affects the probability with which the warehouse gets burnt. These probabilities together with the disutilities of efforts are given in the following table.

		Outcomes and Probabilities		Disutilities
		\$10,000	\$900	$d(e_i)$
Efforts	e_0: negligent	0.80	0.20	0
	e_1: moderately cautious	0.90	0.10	2
	e_2: very cautious	0.95	0.05	4

The owner's preferences are described by a von Neumann and Morgenstern utility function $\sqrt{w} - d(e_i)$ where w is his net wealth and $d(e_i)$ is his disutility of taking action e_i.

(1) Which action is the warehouse owner is going to take?

Suppose that the warehouse owner can buy insurance from a risk-neutral monopolist.

(2) If the insurance monopolist can observe the actions of the warehouse owner, what insurance contract will the monopolist offer? Which action is the warehouse owner going to take? What are the profits of the insurance monopolist?

(3) Suppose that the insurance monopolist *cannot* observe the warehouse owner's actions. Set up the insurance monopolist's problem of designing an optimal insurance policy. Find the optimal insurance contract. What are the profits of the insurance monopolist? Compare this optimal contract with the one you found in (2).