McGill University
Department of Economics
Comprehensive Examination

Microeconomic Theory

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Location: Leacock 424
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Instructions:

- Answer two questions from Part A and both questions from Part B.
- All questions have equal weights.
- Calculators are allowed.
- No notes or texts are allowed.
- This exam comprises 5 pages, including this cover page.

Good luck!

Part A

Question 1
There are only two commodities in the perfectly competitive economy. Consider a consumer whose utility function is given below:

$$u(x) = u(x_1, x_2) = x_1^{1/3} + 3x_2^{1/3} \text{ where } x_1 \geq 0 \text{ and } x_2 \geq 0.$$

Let $p_i > 0$ be the price for good $i = 1, 2$ and $y > 0$ be the consumer's income. Answer the following questions.

1. Find the ordinary (Marshallian) demand function $x(p_1, p_2, y)$.
2. Find the indirect utility function $v(p_1, p_2, y)$.
3. Explicitly verify that the ratio of the derivative of the indirect utility function with respect to prices divided by the derivative with respect to income yields the negative of the ordinary (Marshallian) demand for commodities.

Question 2
Consider a consumer whose utility function is given below:

$$u(x) = u(x_1, x_2) = \ln(x_1 + 1) + x_2 \text{ where } x_1 \geq 0 \text{ and } x_2 \geq 0$$

1. Let $\overline{u} > 0$ be a fixed utility level. Find the compensated (Hicksian) demand function $x^h(p_1, p_2, \overline{u})$, where $(p_1, p_2) \gg (0, 0)$.
2. Find the expenditure function $e(p_1, p_2, \overline{u})$ where $(p_1, p_2) \gg (0, 0)$ and $\overline{u} > 0$.
3. Explicitly verify that the derivative of the expenditure function with respect to prices yields the compensated (Hicksian) demand for commodities.

Question 3

We know that the existence of Walrasian equilibrium is guaranteed under some set of assumptions. Show explicitly that each of the following four economies does not possess an Walrasain equilibrium. Moreover, identify the reason why each economy fails to have an equilibrium. Throughout your analysis, you must assume that equilibrium prices are nonnegative due to the free disposal technology. Furthermore, in all the economies below, you make the assumptions that each consumer's consumption set is $\mathbb{R}^n_+$ where n stands for the number of goods and $\sum_{i \in \mathcal{I}} e^i \gg 0$, where $\mathcal{I}$ stands for the set of consumers.

1. Consider a two person two good exchange economy $(e^i, u^i(\cdot))_{i=1,2}$ in which each individual's preferences are lexicographic and $e^i = (1, 1)$ for each $i = 1, 2$.

2. Consider a two person two good exchange economy $(e^i, u^i(\cdot))_{i=1,2}$ in which $e^i = (1, 1)$ for each $i = 1, 2$; $u^1(x_1, x_2) = -(x_1 - 0.5)^2 - (x_2 - 0.5)^2$, and $u^2(x_1, x_2) = x_1 x_2$.

3. Consider the following production economy with two goods and one consumer. There is one input (good 1) and one output (good 2). The production set is

$$Y = \{(y_1, y_2) \in \mathbb{R}^2 | \; y_2 < \sqrt{-y_1}\} \cup \{0\}.$$

   The initial endowment of the consumer is $(1, 1)$ and this consumer's utility function is $u(x_1, x_2) = x_1 x_2$.

4. Consider the following production economy with two goods and one consumer. The initial endowment of the consumer is $(1, 1)$ and this consumer's utility function is $u(x_1, x_2) = x_1 x_2$. The production set is

$$Y = \{(y_1, y_2) \in \mathbb{R}^2 | \; y_1 + y_2 \leq -3, \; y_1 \leq 0\}.$$

Part B

Question 1

A consumer with initial income $I = 100$ can face either of the 2 states of nature. In state of nature 2 ("accident"), which occurs with probability θ, he incurs a loss with monetary equivalent of $L = 80$. In state of nature 1 ("no accident"), which occurs with probability $(1 - \theta)$, he incurs no loss. There are 2 types of consumers in the market: low-risk type with probability of accident $\theta_L = 0.2$ and high-risk type with probability of accident $\theta_H = 0.5$. The fraction of high-risk consumers in the population is λ. All consumers have the same von Neumann and Morgenstern utility function $u(W) = \ln W$ over income W.

Consumers buy insurance from a perfectly competitive insurance market where all insurance firms are risk-neutral. An insurance contact (between a consumer and an insurance firm) is a pair (p, s), where p is the premium to be paid (by the consumer to the insurance firm) in all states of nature, and s is the amount the firm reimburses to the consumer in case of an accident.

(1) What contracts will the insurance companies offer if they can identify the consumer's type (i.e., information is symmetric)? Will the consumer be fully insured?

(2) Will the insurance companies offer the contracts obtained in (1) if they *cannot* identify the consumer's type (i.e., information is asymmetric)?

(3) Can there exist a pooling equilibrium (in which the consumer buys the same contract regardless of his risk type) under asymmetric information?

(4) Identify the contracts that could comprise a separating equilibrium (in which the consumer buys different contract depending on his risk type) under asymmetric information. Will the consumer be fully insured?

(5) For what values of λ does the equilibrium you identify in (4) exist?

Question 2

Mr. Gates owns MS Co. and hires Mr. Ballmer to manage the firm. Mr. Gates is risk-neutral, but Mr. Ballmer is risk-averse with a utility function of $U(w,e) = \sqrt{2w} - e^2$, where w is the wage he receives and e is the effort he devotes to MS co. Suppose that Mr. Ballmer can either shirk, in which case $e = 1$, or work hard, in which case, $e = 4$. Mr. Ballmer's effort impacts the performance of MS Co. and can generate a gross profit of 1000, 2000, or 3000. The probabilities of the gross profits conditional on effort levels are given in the following table:

effort\profit	1000	2000	3000
$e = 1$	0.6	0.2	0.2
$e = 4$	0.2	0.3	0.5

Mr. Ballmer's reservation utility is 17.

(1) What is the optimal contract when agent's effort is observable (i.e., the *first best* contract)?

(2) Set up Mr. Gates' problem of designing the optimal contract when efforts are unobservable. Find the optimal contract (i.e., the *second best* contract). Compare your results (the feature of the contract, the action induced by this contract, the expected utility of the manager, and the expected profit of the owner) with those for the first best contract.

(3) Suppose that $U(w,e) = 2w - e^2$; therefore, Mr. Ballmer is risk-neutral as well. What is the optimal contract with unobservable efforts?