Evaluation of EEG localization methods using realistic simulations of interictal spikes:

Empirical comparison between Bayesian inference and Maximum Entropy on the Mean

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Overview

- 1. Clinical context: multimodal exploration of the epileptogenic network
- 2. Validation methodology
- 3. Realistic simulation of interictal spike EEG
 - ⇒ Validation of EEG source localization methods
 - ⇒ Empirical Comparison between Bayesian inference and MEM
- 4. fMRI-constrained EEG source localization in epilepsy
 - ⇒ Preliminary results using model comparison
- 5. Conclusion and perspectives





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Epilepsy surgery

• Epileptogenic Zone (EZ): origin and propagation of the seizure ⇒ ictal state

 Irritative Zone (IZ): functional abnormalities between seizures (spikes)
 ⇒ interictal state

• Lesional Zone (LZ) : morphological abnormalities



Organisation as an epileptogenic network





Multimodal exploration of the epileptogenic network



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Validation Methodology



MICCAI 2003 tutorial, Montreal: Validation in Medical Image Processing

Pierre Jannin (IDM Lab, Rennes) Slides available at: http://idm.univ-rennes1.fr/VMIP/miccai2003/





Validation: engineer's viewpoint They develop it ...

- How correct is the output of my software ?
- How robust is my software for the cases I did not take into account ?
- Are there any bugs in my software ?
- How is my software performance compared to other software ? ...





Validation: clinician's viewpoint They use it ...

- How much can I rely on new information provided by the sofware ?
- Does the software improve the health and the quality of life of the patient ?
- Does the software make my daily work easier ?
- Which is the best similar software I can purchase ? ...





Validation-Evaluation Levels

Complexity and diversity of validation

Efficacy of diagnostic imaging systems :

- 1) technical capacity,
- 2) diagnostic accuracy,
- 3) diagnostic impact (i.e. improvement of diagnosis),
- 4) therapeutic impact (i.e. influence in the selection and delivery of the treatment),
- 5) patient outcome (i.e. improvement of the health of the patient),
- 6) societal impact (e.g. cost effectiveness).

from Fryback DG and Thornbury JR, Med. Decis. Making, 11, 1991





Validation Methodology

Validation = Performance evaluation + analysis of evaluation results in a clinical context with a precise objective

- 1. To identify the clinical context
- 2. To specify the validation objective
- 3. Definition of validation criteria:
 - ✓ Internal validity: accuracy
 - ✓ External validity: robustness
 - Reliability: precision
- 4. Definition of validation metrics to assess those criteria (e.g., distance, ROC curves)





Two main approaches

Evaluation based on a comparison with a reference or a Gold Standard:

- Realistic simulation according to the validation objective: absolute Gold Standard
- Approximated Gold Standard: e.g. intra-cerebral EEG recordings
- Evaluation without any available reference:
 - Model comparison or model averaging





Methodology for reference-based validation: comparison with a Gold Standard





Validation Data Sets

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Numerical simulations	 perfect knowledge of the reference: absolute Gold Standard Fine tuning 	realism of simulated data
Realistic simulations from clinical data sets	 perfect knowledge of the reference: absolute Gold Standard Better realism 	realism of simulated data
Physical phantoms	 the whole image acquisition set up is taken into consideration 	 few of them are multimodal approximated Gold Standard from dedicated protocols
Cadavers	realistic data	 approximated Gold Standard no functional or metabolic information
Reference data sets (Vanderbilt project, Visible Human)	 facilitate comparison of validation results 	 "hard" and unusual cases patient information update
Clinical data sets	• best realism	 approximated Gold Standard or even no Gold Standard



Requirements for Model definition and Validation Methodology

- Both require comparison
- Both deal with some prior information either about the solution or about the overall objective of the method
- Both require an estimation of the uncertainties





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EEG source localization : forward problem, inverse problem

Forward problem: physical model definition



Brain model (G) + source model (J)





Generated signal on the scalp (M)

• Inverse problem: estimation of the EEG sources (J) from a measured signal (M)















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EEG source localization: Equivalent Current Dipole (ECD) vs Distributed source

EEG source model is written as: M = GJ + E

Dipôle equivalent

Equivalent Current Dipole (ECD):

Brain Sources

Model amplitude

Noise

- non linear problem: G?, J?
- number of sources ?
- what is an ECD ?

Signal

Distributed source method :

- Anatomical constraint
 ⇒ linear problem: J?
- p = 10³ sources ~ n= 10² sensors
 - ⇒ ill-conditioned problem
- regularization needed





The EEG/MEG forward problem: the distributed sources model



Example of EEG source localization of an interictal spike

Signal + Scalp Potentials

Single ECD



- 60 - 40 - 20 - 0 - -20 - -20 - -40 - -60 Most likely ECD Combinations (Bénar et al. IEEE TBME 2005)



Distributed source (e.g., LORETA)





Gold Standard = Depth recordings



fMRI negative resp.



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EEG source localization of interictal spikes

- Depth recordings showed that interictal spike generators are rarely focal (Merlet et al. Clin. Neurophys. 1999)
- ECD are thought to represent the center of mass of such generators
- A minimum brain activated area of 6 cm² is needed to generate a spike on the scalp (Ebersole J. Clin. Neurophys. 1997)
- Spike generators may be quite more extended than 6 cm² (a whole lobe)

⇒ What is the behaviour of distributed source localization methods in the presence of extended sources ?





Validation model

- Clinical context: source localization of EEG interictal spikes
- Validation objective: « Are distributed source localization methods able to localize extended sources with good detection accuracy (i.e. > 80% good detections) ?»
- Reference: absolute Gold Standard provided by simulations of EEG interictal spikes
- Validation criteria: detection accuracy
- Validation metric: Area Under the ROC Curve (AUC) as a detection accuracy index



Validation of distributed source localization methods using realistic simulations

Anatomical and Functional Models: Gold Standard =Jtheo Physical model to generate scalp potentials G Source Localization on simulated signals

Evaluation of the detection accuracy : Area under the ROC curve (AUC)

ROC Curves

1 – specificity = false negative rate

Simulation of realistic EEG interictal spikes

The EEG forward problem: estimation of G Head models and resolution approaches

a : spherical model (analytical solution)
b : realistic surface model (BEM) → BrainStorm
c : realistic volume model (FEM)

Source localization methods to be evaluated: 1. Bayesian framework (1/2)

- Bayes' rule: $Prob(\mathbf{J}|\mathbf{M}; \sigma_E, \sigma_J) = \frac{Prob(\mathbf{M}|\mathbf{J}; \sigma_E).Prob(\mathbf{J}|\sigma_J)}{Prob(\mathbf{M}|\sigma_E, \sigma_J)}$
- Model of the noise E pdf: Gaussian(0,Var[E] = $\sigma_E^2 I_n$)
- Model of the prior source J pdf: Gaussian(0,Var[J] = σ_J^2 (W W^T)⁻¹)

kelihood:
$$Prob(\mathbf{M}|\mathbf{J};\sigma_E) = \frac{1}{(2\pi \ \sigma_E^2)^{n/2}} \ exp\left(-\frac{1}{2\sigma_E^2}\|\mathbf{M} - \mathbf{G} \cdot \mathbf{J}\|^2\right)$$

$$Prob(\mathbf{J}|\sigma_J) = \frac{1}{(2\pi)^{p/2}} \frac{1}{|\sigma_J^2(\mathbf{W}\cdot\mathbf{W}^T)^{-1}|^{1/2}} exp\left(-\frac{1}{2\sigma_J^2}\|\mathbf{W}\cdot\mathbf{J}\|^2\right)$$

• MAP estimate:

• Prior pdf:

$$\hat{\mathbf{J}}_{\sigma_E,\sigma_J} = argmin_{\mathbf{J}} \left[\|\mathbf{M} - \mathbf{G} \cdot \mathbf{J}\|^2 + \alpha \|\mathbf{W} \cdot \mathbf{J}\|^2 \right]$$
 with $\alpha = \frac{\sigma_E^2}{\sigma_T^2}$

$$\hat{\mathbf{J}}_{\sigma_E,\sigma_J} \cdot = (\mathbf{G}^T \cdot \mathbf{G} + \alpha \mathbf{W}^T \cdot \mathbf{W})^{-1} \cdot \mathbf{G}^T \cdot \mathbf{M}$$

Source localization methods to be evaluated: 1. Bayesian framework (2/2)

- We evaluate the MAP estimates with commonly used prior
- The hyperparameter α was estimated using the empirical approach of the L-curve
- Minimum Norm (MN): W = I_p
 - ⇒ H: all sources are independent and have the same power
- Weighted Minimum Norm (WMN): W(i,i) = 1 / MSP(i)

MSP : Multivariate Source Prelocalization (Mattout, Neuroimage 2005)

- ~ probability of activation of each source « from the data »
- H: all sources are independent and have a power linked to the probability of activation of each source
- Bayesian LORETA1: W = ∆ : discrete Laplacian computed on the cortical mesh
 ⇒ H: the mean activity of a source is linked to its spatial neighborhood (spatial smoothness)
- Bayesian LORETA2 : Var[J] = σ_J² (Δ Δ^T + α_{MN} I_p)⁻¹) : additional MN regularization to limit numerical instabilities due to the computation of Δ on a fully connected cortical mesh

Source localization methods to be evaluated: 2. Maximum Entropy on the Mean (MEM) (1/3)

- Estimating J as a realization of a random variable is equivalent to estimating its distribution: dp(j) = P(J = j)
- Principle of the MEM: estimating dp that maximizes the missing information, given the data M
- Prior information: definition of a reference distribution : dµ
- Solution of the form: dp(j) = f(j) dμ(j), where f is a μ-density to be found such that it explains the data in average (noise being zero mean):

$$\mathbf{M} = \int \mathbf{G} \cdot \mathbf{j} \ f(\mathbf{j}) d\mu(\mathbf{j})$$

• We will note C_M the set of all distribution dp verifying such constraint

Source localization methods to be evaluated: 2. Maximum Entropy on the Mean (MEM) (2/3)

Maximum Entropy of the Mean solution:

$$d\hat{p} = \arg\max_{dp \in \mathcal{C}_{M}} H_{\mu}(dp) \quad \text{with} \quad H_{\mu}(dp) = -\int f(\mathbf{r}) \cdot \log(f(\mathbf{r})) d\mu(\mathbf{r})$$
$$\mathbf{M} = \int \mathbf{G} \cdot \mathbf{j} \ f(\mathbf{j}) d\mu(\mathbf{j}) \quad \Longrightarrow \quad \mathbf{C}_{M} \ dp^{*}(\mathbf{j}) \quad \underbrace{\mathbf{J}}_{\mathbf{j}} \quad \underbrace{\mathbf{J}}$$

 MEM Solution: unique solution, optimization in a n dimension space (n: number of sensors) (see Amblard IEEE TBME 2004 for proof)

$$\hat{\mathbf{J}}_{MEM} = \nabla F_{\mu}(\xi)_{|\xi=\mathbf{G}^T\tilde{\lambda}}$$
 with $F_{\mu}(\xi) = \log \int exp(\xi^T \mathbf{j}) d\mu(\mathbf{j})$

and $\tilde{\lambda} = \operatorname{argmax}_{\lambda} \left(\lambda^T \mathbf{M} - F_{\mu}(\mathbf{G}^T \lambda) - \frac{\sigma_E^2}{2} \lambda^T \cdot \lambda \right)$

Source localization methods to be evaluated: 2. Maximum Entropy on the Mean (MEM) (3/3)

Definition of Prior Information : $d\mu$

$$d\mu(\mathbf{j}) = \Pi_{k=1}^{K} \left[(1 - \alpha_k) \delta(\mathbf{j}_k) + \alpha_k \, \mathcal{N}(\mu_k, \Sigma_k)(\mathbf{j}_k) \right] d\mathbf{j}$$

- K cortical parcels assumed to be independent (K ~ n)
- Each parcel is associated to an hidden state variable Si (Si = 1 : parcel active)
- If a parcel is active (Prob(Si =1) = α_i), a gaussian distribution of J is assumed N(μ_i , Σ_i)
- MEM1: $\mu_i = 0$, α_i from the MSP
- MEM2: μ_i from MN solution, α_i from the MSP
- MEM estimate:

$$\hat{\mathbf{J}}_{MEM}^{k} = \hat{\alpha_{k}} \left[\mu_{k} + \Sigma_{k} \mathbf{G}_{k}^{T} \tilde{\lambda} \right] \text{ with } \hat{\alpha_{k}} = \frac{\alpha_{k}}{\alpha_{k} + (1 - \alpha_{k}) \exp(-F_{\mu,k}(\mathbf{G}_{k}^{T} \tilde{\lambda}))}$$

$$F_{\mu,k}(\mathbf{G}_k^T\tilde{\lambda}) = \mu_k^T \mathbf{G}_k^T \tilde{\lambda} + \frac{1}{2}\tilde{\lambda}^T \mathbf{G}_k \Sigma_k \mathbf{G}_k^T \tilde{\lambda} \qquad F_{\mu}(\mathbf{G}^T \lambda) = \sum_k \log\left[(1 - \alpha_k) + \alpha_k \exp(F_{\mu,k}(\mathbf{G}_k^T \lambda))\right]$$

Source localization methods to be evaluated: 1. Methods deduces from Bayesian framework

Source localisation	Optimization	Assumption
Method	Function	for regularization
Minimum norm (MN) Hamalainen et al. Med Biol. Eng. Comput. 94 Weighted minimum norm (WMN) Mattout et al. ISBI 2001	Min (M-GJ ² + α J ²) Min (M-GJ ² + α W J ²)	Minimum energy solution
Low resolution electromagnetic tomography (LORETA) Pascual-Marqui et al. Int. J. Psychophys. 94	Min $ \Delta \mathbf{J} ^2$ under the constraint $\mathbf{M} = \mathbf{G}\mathbf{J}$ Δ : discrete spatial Laplacian	Maximum of spatial smoothness solution
Maximum entropy of the	Max Entropy(P _J ; P _{prior})	Solution
mean (MEM)	under the constraint M = GJ	with less assumption
Amblard et al. IEEE TBME 2004	on average	regarding missing data

Validation Metric to assess detection accuracy: Area under the ROC curve

- Receiver Operating Characteristic (ROC) Curve
 ⇒ to study detection accuracy
- Generation of binary maps of activation :
 - Estimated $\hat{J} \Rightarrow \sqrt{(\hat{J}^2)} / \sqrt{(\hat{J}max^2)}$
 - Gold Standard : Jtheo = 1 if dipole activated, 0 otherwise
- Construction of the ROC curve
 For each threshold t varying
 between 0 and 1, we estimate:

	Ĵ < t	Ĵ > t
Jtheo = 0	True Negative	False Positive
Jtheo = 1	False Negative	True Positive

– Sensitivity = True Positive Rate

Specificity = True Negative Rate_

ROC: sensitivity = f(1-specificity

Validation Metric to assess detection accuracy: Area under the ROC curve

ROC Curves

Area under the ROC Curve (AUC) =

probability of good detection when the same amount of points with and without activation are presented to the observer

Biased estimation: # active sources << # inactive sources

 Adaptation to the context of distributed sources: randomly drawing the same number of fictive sources as the number active sources

• fictive sources drawn in the close neighborhood: AUC.close

s fictive sources drawn in local maxima located far from the patch: AUC.far

Validation Results: 2nd order spatial extent (5 cm²)

Gold Standard:

Temporo-Radial Source

Validation Results: 4th order spatial extent (14 cm²)

Gold Standard: Temporo-Radial Source

Gold Standard:

Temporo-Tangential Source

Validation Results: 7th order spatial extent (36 cm²)

Gold Standard: Temporo-Radial Source

Gold Standard:

Temporo-Tangential Source

Validation results: summary

Distributions of AUC

(all locations + all spatial extents)

Distributions of the min Euclidian distance

between the global maximum of energy and the simulated source

- Most accurate methods (AUC > 0.8): LORETA2 and MEM2
- Less robust method: LORETA
- Less false positive rate: WMN 38

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Simultaneous EEG-fMRI acquisition: setup

Multiple problems in recording the EEG inside the scanner: ballistocardiogram, movement, gradient

40 Gotman, Bénar and Dubeau, *J Clin Neurophys* 2005

Simultaneous EEG-fMRI acquisition: data analysis

"Scénarios catastrophe": motivations.

1. Divergence entre l'activité bioélectrique (EEG) et l'activation métabolique (IRMf BOLD).

vue par l'EEG et par l'IRMf

vue par l'EEG mais pas (ou mal) par l'IRMf

2. Un a priori statique pour un réseau dynamique.

A priori assumptions.

$$p(\mathbf{J} \mid \epsilon, \sigma^2, H) \sim \exp\left(-\frac{\epsilon^2}{2\sigma^2} \sum_{j=1}^t \left\|\mathbf{L}^{(H)}\mathbf{J}_j\right\|^2\right)$$

H0 "a priori, sources are independent and have the same power $\mathbf{L}^{(H_0)} = \mathbf{I}_n$

H1: "a priori, sources are independent and have the a power linked to the fMRI activation map"

$$\mathbf{L}^{(H_1)} = \left(diag\left(f\left(\overrightarrow{\mathbf{Z}} \right) \right) \right)^{-\frac{1}{2}}$$

Hierarchical Bayesian Model (cf. J. Daunizeau)

$$p(\mathbf{J} \mid \sigma^{2}, \epsilon, \mathbf{M}, H) = \frac{p(\mathbf{M} \mid \mathbf{J}, \sigma^{2}) \cdot p(\mathbf{J} \mid \sigma^{2}, \epsilon, H)}{p(\mathbf{M} \mid \sigma^{2}, \epsilon, H)}$$

$$p(\sigma^{2}, \epsilon \mid \mathbf{M}, H) = \frac{p(\mathbf{M} \mid \sigma^{2}, \epsilon, H) \cdot p(\sigma^{2}, \epsilon)}{p(\mathbf{M} \mid H)}$$

$$p(H_{i} \mid \mathbf{M}) = \frac{p(\mathbf{M} \mid H_{i}) \cdot p(H_{i})}{p(\mathbf{M})}, \ i = 0, ..., d$$

Inference by Maximum a Posteriori

1. Parameters

*

$$\hat{\mathbf{J}} = \left(\mathbf{G}^T \mathbf{G} + \epsilon^2 \mathbf{L}^{(H)T} \mathbf{L}^{(H)} \right)^{-1} \mathbf{G}^T \mathbf{M}$$

2. Hyperparameters

$$\hat{\boldsymbol{\epsilon}} = \arg_{\boldsymbol{\epsilon}} \left[\epsilon^{nt-2} \mid \boldsymbol{\Sigma}(\boldsymbol{\epsilon}, H) \mid^{-\frac{t}{2}} \left(tr \left[\mathbf{M}^T \mathbf{M} \right] - tr \left[\hat{\mathbf{J}}^T \boldsymbol{\Sigma}(\boldsymbol{\epsilon}, H) \hat{\mathbf{J}} \right] \right)^{-\frac{pt}{2}} \right]$$

3. Pertinence of the a priori model

$$p(\mathbf{M} \mid H_i) = K \mid \mathbf{L}^{(H_i)} \mid^t I_i$$

Validation model

Clinical context: source localization of EEG interictal spikes
constrained by fMRI results

• Validation objective: « Is it possible to quantify the pertinence of an informative prior regarding the EEG data only?"

 Reference: absolute Gold Standard provided by realistic simulations: simulation of source of EEG interictal spikes and fMRI activation map: fMRI map= perturbation of the simulated EEG sources

- Validation criteria: Pertinence of using an informative prior
- Validation metric: assessing this pertinence with or without Gold Standard

Validation metrics

→ Validation metric without Gold Standard (GS): Bayes factors

$$\alpha = \log\left(\frac{p(H_1|\mathbf{M})}{p(H_0|\mathbf{M})}\right)$$

 $\rightarrow \alpha > 0$: the informative prior H1 is more pertinent than the non informative one H0, regarding the EEG data only !

→ Validation metric with Gold Standard (GS): Sum of Square Errors (SSE), Area under the ROC curve (AUC)

 $\gamma = \log(SSE1/SSE0), \gamma > 0$: the estimate using the informative prior H1 generate more errors than the non informative one H0 $\beta = \log(AUC1/AUC0), \beta > 0$: the estimate using the informative prior H1 has better

detection accuracy than the non informative one

Validation data sets:

\rightarrow 50 EEG extended sources simulated: Jeeg

- Perturbation of the fMRI map by noise: Zfmri = (Jeeg + noise)²
- Discrepancy between the location of the EEG source Jeeg and the fMRI activation Zfmri, from a distance d (d = neighborhood order from 1 to 13)

Effect of noise perturbation: Zfmri = (Jeeg + noise)²

α (without GS) vs β (with GS)

 α (without GS) vs γ (with GS)

Daunizeau, Grova et al, IEEE TSP, 2005, in press

Effect of the discrepancy (distance d) between EEG source and fMRI map

α (without GS) vs d = log(d/ 2cm) γ (with GS) vs d = log(d/ 2cm)

Daunizeau, Grova et al, IEEE TSP, 2005, in press

Preliminary results on real epileptic data: Case 1

Average Spike on the scalp (42 sensors)

Interpolation of the fMRI map on the cortical surface

H0: noninformative prior

H1: informative prior (fMRI)

t2

Incomplete Gold Standard = Intra-cerebral recordings

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t1

Preliminary results on real epileptic data: Case 2

Average Spike on the scalp (43 sensors)

fMRI positive response (T>3.1)

L

Interpolation of the

fMRI map on the

cortical surface

t1

H0: non-informative prior

H1: informative prior (fMRI)

⇒ α=0.69

Incomplete Gold Standard = Intra-cerebral recordings

Conclusion and perspectives

- Both validation and prior model should be defined regarding a precise clinical objective
- Realistic simulations provide an ideal framework to study many parameters of source localization methods
- Our evaluation study: pro's and con's for each method (e.g., WMN, LORETA, MEM) : they should be compared
- Hierarchical Bayesian Model: model selection and model comparison
- The link between fMRI and EEG sources is difficult and should be considered with caution even more in epilepsy: Bayesian model comparison may help !
- Validation on more real data is required

Measuring the pertinence of the model within the MEM approach ?

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