First Principles Support for First Order Flotation Kinetics

Premise: rate of removal of particles is the product of number of particles collected per bubble times the number of bubbles per second passing through the cell. We examine derivations from two sources.

A. Jameson/Nam/Young (1977)

Based on Figure 1, a stepwise derivation is outlined.

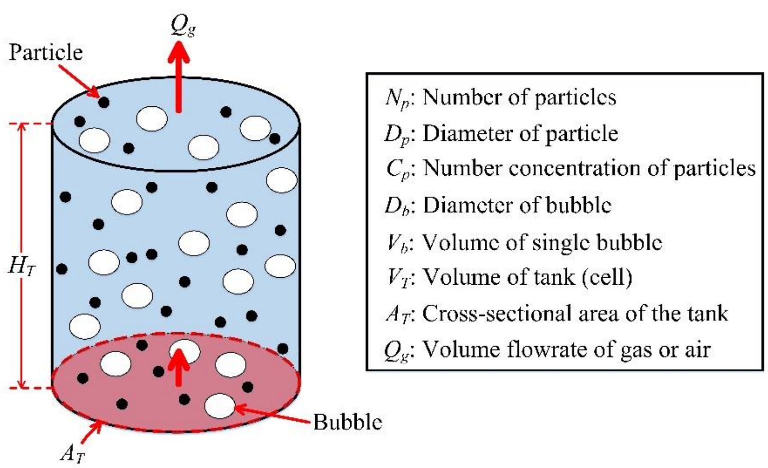

Figure 1. Summary of conditions/definitions for derivation using Jameson/Nam/Young approach

1. The number of particles encountered per bubble of diameter (D_b) rising through the cell of height (H_T) is

$$\frac{\pi}{4}\ (D_b + D_p)^2 H_T C_p$$

   where C_p is the number concentration of particles, $C_p = N_p/V_T$

2. Since not all particles encountered are collected, we introduce a 'collection efficiency' (or probability) E_K and we also simplify by assuming $D_p << D_b$ to give

$$\frac{\pi}{4}\ D_b^2\, H_T\ E_K\ C_P$$

3. For a volumetric gas flowrate Q_g the number of bubbles each of volume V_b produced per unit time per unit of cell volume V_T (V_T is needed as particle concentration is number per unit volume, N_p/V_T)

$$\frac{Q_g}{V_b V_T}$$

4. Therefore, rate of removal of particles, dN_p/dt, becomes (note negative sign as C_p is decreasing):

$$\frac{dN_P}{dt} = -\frac{\pi}{4} D_b^2\ H_T\ E_K \frac{Q_g}{V_b V_T}\ C_P$$

5. This equation is of the form:

$$\frac{dN_P}{dt} = -k\ C_P$$

which supports the first order assumption, where the first-order rate constant, *k* is:

$$k = \frac{\pi}{4}\ D_b^2\ H_T\ E_K \frac{Q_g}{V_b V_T}$$

A check on units will reveal that k has the appropriate unit of 1/time (e.g., 1/s)

B. King (2012)

For this derivation rather than cell height, bubble (and particle) velocity is introduced (Figure 2) otherwise conditions are as in Figure 1. The same stepwise approach to the derivation is used.

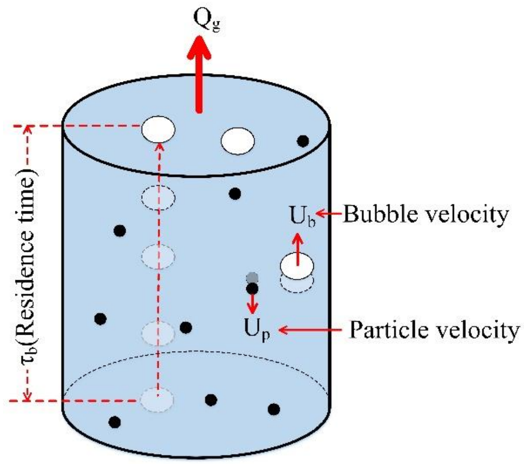

Figure 2. Conditions for derivation using King approach (see also Fig. 1)

1. The number of particles encountered per bubble with residence time (τ_b) in the cell is (upwards is positive)

$$\frac{\pi}{4}\,(D_b + D_p)^2 (U_b - U_p)\tau_b C_p$$

2. Again simplifying ($D_p < D_b$; $U_p < U_b$) and noting that not all particles encountered are collected by introducing E_K we arrive at number of particles collected per bubble:

$$\frac{\pi}{4}\,D_b^2\,U_b\,E_K\,\tau_b C_p$$

3. The rate of removal is given by multiplying by number of bubbles per unit time per unit cell volume:

$$\frac{dN_P}{dt} = -\frac{\pi}{4} D_b^2\,U_b\,E_K\,\tau_b\,\frac{Q_g}{V_b V_T}\,C_p$$

This again supports the first order kinetics, with k given by

$$k = \frac{\pi}{4} D_b^2 U_b \tau_b E_K \frac{Q_g}{V_b V_T}$$

Some observations:

1. The two derivations are the same since

$$U_b \tau_b = H_T$$

2. The Jameson/Nam/Young result can be converted to relate to gas superficial velocity (J_g) and consequently relate to bubble surface area flux (S_b), which has experimental support (Gorain et al., 1997). The conversion is as follows:

Substitutions:

$$V_b = \frac{1}{6} \pi D_b^3$$

$$\frac{H_T}{V_T} = \frac{1}{A_T}$$

$$J_g = \frac{Q_g}{A_T}$$

Therefore:

$$\frac{dN_P}{dt} = -\frac{3}{2} \frac{J_g}{D_b} E_K C_P$$

$$k = \frac{3}{2} \frac{J_g}{D_b} E_K$$

And, since:

$$S_b = \frac{6 J_g}{D_b}$$

Then:

$$k = \frac{1}{4} S_b E_K$$

3. The King result can be converted to relate to gas holdup (ε_g), and linear dependence on gas holdup has experimental support (Finch et al., 2000; Hernandez et al., 2003). The conversion is to substitute for V_b and τ_b, where:

$$\tau_b = \frac{V_T \, \varepsilon_g}{Q_g}$$

Therefore:

$$\frac{dN_P}{dt} = -\frac{3}{2} \, U_b \frac{\varepsilon_g}{D_b} \, E_K \, C_P$$

$$k = \frac{3}{2} \, U_b \frac{\varepsilon_g}{D_b} \, E_K$$

4. The two results must still be the same; that is, they are different forms of the same relationship. This means that

$$J_g = U_b \, \varepsilon_g$$

Which is the case.

5. Bubble surface area flux (S_b) replaces the J_g/D_b ratio. Similarly, the ratio ε_g/D_b can be replaced by the bubble surface area per unit tank volume, A_b. From the derivation of A_b (see derivation of S_b and A_b on the website) the King result can be re-written in terms of A_b:

$$A_b = \frac{6 \, \varepsilon_g}{D_b}$$

$$k = \frac{1}{4} \, U_b \, A_b \, E_K$$

Since the two results are the same, then

$$U_b A_b = S_b$$

as King notes.

Again, we might anticipate that either an approximately linear dependence of rate constant on either S_b or A_b exists, provide U_b is not a significant factor.

6. From these results we might anticipate that k is linearly related to either S_b or A_b. The limited experimental evidence is that k is related to S_b and ε_g (Finch et al., 2000; Hernandez et al., 2003), the latter case seeming to argue against linearity with A_b, although dependence n A_b. does not seem to have been tested. One reason the linearity with A_b.is not found is the impact of U_b noting that $U_b A_b = S_b$.

7. The result shows k is a complex function of operating conditions (D_b, Q_g. E_K) and particle properties (D_p, E_K). This does not detract from the support for the first order hypothesis. It does remain the case that in the derivations it is understood that 'particle' refers to a given size and hydrophobicity; that is, a unique particle class. Nevertheless it remains common to apply a single k to a range in classes, as otherwise this requires that either the rate constant has to be determined for each class or a function capturing the distribution of rate constants is required to treat the range in particle classes in actual practice. Several k-distribution functions have been proposed.

8. For first order to apply it is necessary that enough bubble surface be available to capture the eligible encountered particles. If this is not the case, that the bubble is "loaded" with particles, then recovery is carrying capacity limited, which is effectively a zero-order rate process. Estimates of carrying capacity are available (another topic).

9. The results left E_K without comment. It is an efficiency (or probability) of collection (capture) factor. From the sub-processes occurring in particle collection by bubbles in flotation, E_K is the product of collision efficiency (E_C), attachment efficiency (E_A) and stability of attachment, usually expressed as (1 - E_D) where E_D is probability of detachment. That is:

$$E_K = E_C E_A (1 - E_D)$$

Fundamental flotation models aim to derive expressions for these three probabilities, an area of continuing research.

References:

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Gorain, B.K., Franzidis, J.P., Manlapig, E.V. 1997. Studies on impeller type, impeller speed and air flow rate in an industrial scale flotation cell. Part 4: Effect of bubble surface area flux on flotation performance, *Minerals Engineering*, 10(4), 367-379.

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