Important information

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Lectures: Tuesdays and Thursdays: 02:35pm - 03:55pm, SH688 room 495
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Course overview

There is a long-lasting relationship between philosophy and mathematics, starting at least with the Greeks and continuing in contemporary philosophy. This relationship has taken various forms. First, there is the fact that mathematicians, like Frege and Poincaré, have made important philosophical contributions. Conversely, philosophers have made important contributions to mathematics, e.g. Descartes and Leibniz. Philosophy has also been called upon by mathematics, especially in periods of crisis or important mathematical change, for the justification or the interpretation of mathematical developments. The so-called *foundational crisis* of mathematics is usually taken to be a good example of that. Finally, many philosophical questions regarding the specificity of mathematics have been raised—questions regarding the nature of the objects studied by mathematics, the methodology of mathematics, or the relation of mathematics to empirical sciences and the in/dispensability of the former to the latter—and philosophers of mathematics have been busy proposing answers to these questions.

This course will provide an overview of philosophy of mathematics, with an emphasis on the last 100 years or so, and will enable students to engage with contemporary research in philosophy of mathematics. The course is divided in four roughly equal parts, covering 1) central concepts and figures, 2) early twentieth century and the foundational crisis, 3) later twentieth century and answers to Benacerraf's challenge, and 4) recent work and new directions in the philosophy of mathematics. Throughout the semester, we will be covering the following topics and questions[1].

In the first lectures of the session, with the help of examples from elementary mathematics, we will introduce ourselves to basic concepts and to some historical conceptions of mathematics.

1. **Introduction: what does philosophy have to say about mathematics?**

   - What is an axiomatic system? What is a proof? What is a set?
   - What is *infinity*? What is *generality*? What is *abstraction*?

   We will be looking at some examples from diverse areas of mathematics, like geometry (paper-and-pencil proofs), arithmetic (algorithms and prime numbers), combinatorics (graph theory) and at contributions by important historical figures like Euclid, Plato, Euler or Kant.

We will then look at some answers to the general question: what is the nature of mathematics?

[1] A more detailed schedule will be provided at the beginning of the semester.

2. **Early twentieth century and classical foundational positions**

   • Is mathematics just logic in disguise?

   • Is mathematics just symbol manipulation?

   • Is mathematics just a creation of the mind?

   We will be covering the "big three", namely Frege and Russell's logicism, Hilbert's program and formalism, and Brouwer and Heyting's intuitionism.

We will then study ontological and epistemological questions regarding the subject-matter of mathematics and our access to the objects it studies.

3. **Late twentieth century views on truth, objects, and applicability**

   • What is the subject-matter of mathematics? What are mathematical objects? Do numbers exist?

   • Do mathematical statements mean anything? How do we know mathematical statements? What is the right semantics for mathematics?

   • Can there be science without mathematics? (How) can mathematics tell us something about the world? What is it about the world that makes mathematics applicable?

   We'll be looking at Benacerraf's seminal papers and some reactions to it, like arguments for the indispensability of mathematics (e.g. Putnam) or for nominalist ontologies (e.g. Field).

Finally, we will cover topics that go beyond the traditional questions studied in philosophy of mathematics.

4. **New directions: challenging foundations and mathematical practice**

   • Is progress in mathematics cumulative? Are there mathematical revolutions? How should we characterize progress in mathematics?

   • Can a picture be a proof? Can proofs be beautiful? Are there acceptable gaps in proofs? How can different proofs prove the same theorem?

   • What are notations good for? What is a good notation? Are (some of) our mathematical capacities similar to those of other species? How do metaphors play on our mathematical conceptualizations?

   We'll be reading papers on historical and methodological issues (e.g. Andersen, Dutilh Novaes, Kitcher, Lakatos), as well as work on mathematical cognition (e.g. De Cruz, Lakoff & Nuñez).

Required reading material

All readings for the course will be available on *myCourses* or through McGill's library website.

Evaluation

1. Weekly discussion questions and notes (15%)
2. A critical summary of a contemporary research article (25%)
3. Outline of final essay (10%)
4. Final paper (40%)
5. In-class participation (10%)

Prerequisites

Students should have taken the course *Introduction to deductive logic* (Phil 210) or an equivalent course. Although helpful, no other courses in philosophy or in mathematics are required. Examples discussed in the course will be self-contained.

McGill Policy Statments

Language of Submission

"In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded."

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