Phil 210: *Introduction to Deductive Logic* *Autumn 2024*

MICHAEL HALLETT *E-mail*: michael.hallett@mcgill.ca

Office Hours: *To Be Announced.* (These will probably be over Zoom.)
Look for announcements on *myCourses* concerning *all* Office Hours.

Lectures: Mondays, Wednesdays, 13.35–14.25 ADAMS AUDITORIUM.

There will conferences at this same time on Friday, but other conferences too. *Look for further announcements*.

Conference: 1 hour weekly— Times TBA (most often, Thursday or Friday)

Summary of Material. This course examines the main elements of DEDUCTIVE LOGIC, the modern form of the discipline which has traditionally studied correct forms of inference and reasoning. Modern deductive logic (which stems from the late 19th c.) is primarily concerned with correctly deducing a CONCLUSION from given PREMISES, thus with what is often called VALID INFERENCE/ARGUMENT or LOGICAL CONSEQUENCE or (informally) '[conclusion] following (logically) from [premises]'. The key ideas are introduced first using a simplified language called TFL, which abbreviates 'truth-functional logic'. (NB: This language is *not* English, though we begin with English examples, but is in fact a family of *artificial* languages, which reflects a fragment of (English) linguistic behaviour we're familiar with.) These languages take as a base some elementary (or atomic) *sentences*, and then incorporates a few simple ways to *connect* sentences (connectives), to form more *complex sentences*; it is assumed that all these sentences can then be used first to *state* or *assert* things (PROPOSITIONS, as they are sometimes called), and also used to form what we call ARGUMENTS or INFERENCES, with sentences as premises, and a sentence as a conclusion), thus certain chains of sentences.

With this as basis, we introduce the notion of TRUTH-VALUATIONS, systematic ways of assigning truth-values (T/F, standing for 'true'/'false') to sentences of the language. This notion is then used to characterise VALIDITY OF INFERENCE for this language (there is no truth-valuation which makes the premises all true and the conclusion false), and other important notions, most notably of a sentence being true under all valuations (a TAUTOLOGY, sometimes called a *truth-functionally valid sentence*), the TFL EQUIVALENCE of sentences (same truth-value under all valuations), and the SATISFIABILITY of sets of sentences (there is a valuation making all the sentences of the given set true). We explore the central links between all these. And we will also set-up the system of TRUTH-TABLES for a finite number of sentences, which will of course include arguments. This will provide a means to *check the validity and invalidity of sentences and of arguments*, a way to check whether finite sets of sentences are SATISFIABLE, or whether a sentence of TFL is EQUIVALENT to another.

From here, we proceed to develop a PROOF or DEDUCTION system for TFL, i.e., a way of *deriving* step-by-step logical consequences from given TFL sentences taken as a starting point. For this, we use a system of what is called NATURAL DEDUCTION, the FITCH SYSTEM. The proof/deduction system in effect is built on a small number of clearly valid inferences (many of them reflecting the way we 'naturally' reason), and transforms these into rules of inference, which then, working together, can then be shown to yield *all* valid inferences. This is called the *Completeness of the Deduction System*. We will explain the importance of this, but we will not embark on a proof of it in any detail, since this is complex and intricate.

In the second part of the course, we consider more complex artificial languages, the language(s) for FOL (standing for first-order logic); these languages account for subtleties in sentences that TFL is blind to, like 'there at most three cats' (or 'exactly three', or 'at least three').

The FOL languages are made up as follows. We first introduce NAMES for objects and then a limited (finite) number of what we call PREDICATES, which are in effect *names for a finite number of properties of objects, and of relations between objects*. (One special relation is that of the IDENTITY between objects, so $a = b$.) We then extend this to the use of VARIABLES for objects, and what are called QUANTIFIERS over these variables (this is why FOL is sometimes known as QUANTIFICATIONAL LOGIC). This together gives more refined and therefore more complex means of expression, allowing us to deal with *generality* and with various *existence* claims (such as the ones about the cats above). We can now form sentences which speak of *all* students and at the same time speak of specific *specific* students, as in the simple example 'Jessica is taller than John, and taller than all the students in the class'. As the use of 'and' here indicates, we use the same connectives as were introduced earlier to add more complexity, both within and between sentences. However, this more complex way of forming sentences requires a more complex way of assigning truth-value to sentences. For this, we introduce the notion of an INTERPRETATION of an FOL language, and then see that interpretations induce truth-values on sentences. Given this, we can take over our earlier characterisation of the basic concepts, e.g., valid inference, equivalence of sentences, sentences being always true, etc. almost exactly as before. It is just that 'truth of sentences' is not a primitive notion anymore, as it was in TFL languages.

Following this, we then adopt a correspondingly more complex system of NATURAL DEDUCTION FOR FOL which extends the system we used for TFL, but is more elaborate because we now have to deal with rules for quantification and for identity. With this system, we can prove many things which we would not be able to prove in the TFL system. Moreover, we have a *Completeness Proof* similar to that for the TFL system, namely that a sentence can be validly inferred from some other sentences if, and only if, it can be deduced from them in the natural deduction system. But again, we can only gesture at the result and emphasise its importance.

What Will You Learn in This Course?

The successful PHIL 210 student should be able to:

- LEARN TWO ARTIFICIAL LANGUAGES, the languages of TFL and FOL, and develop the ability to translate (some) natural language sentences into these formal, artificial languages, as well as the ability to parse the meaning of sentences in these artificial languages. In short, you will learn a substantial amount about what philosophers refer to as *logical form*.
- Concerning the FOL languages in particular, the student will LEARN HOW TO RECOGNISE AND UNDERSTAND QUANTIFIERS AND QUANTIFIED SENTENCES, and learn the importance of the *order* of quantifiers, e.g., the difference between 'Every person has a favourite actor' (which has a 'for all/there exists' form), and 'Some actor is everyone's favourite' (which has a 'exists/for all' form').
- UNDERSTAND KEY LOGICAL CONCEPTS: You will learn the basic notions of first TRUTH-VALUATION and later INTERPRETATION, then about the basic semantic concepts (validity, entailment and logical equivalence) for the languages presented which rest on these notions. You will become adept at identifying patterns of valid argumentation.
- REASON WITH TRUTH-TABLES: You will learn to construct and interpret truth-tables so as to evaluate certain properties of sentences and arguments in TFL (determine whether a sentence is a tautology, a contradiction or a contingent sentence, whether a set of sentences are jointly satisfiable, whether an argument in the language of TFL is valid or invalid).
- In particular, you will learn how to use truth-tables to produce COUNTEREXAMPLES for invalid arguments in TFL, that in an invalid argument, the premise can be true in the same circumstances where the conclusion is false. Later you will learn how to design interpretations which provide simple counterexamples for invalid arguments in the languages of FOL.

- PROOF TECHNIQUES: You will learn a particular system of natural deduction called the Fitch system, which we use for both the TFL languages and (extended) for the FOL languages. You will learn to construct derivations from premises to conclusion, how to apply the appropriate rules in different circumstances. You will see that (and why) the derivations so built *guarantee* that there can be no counterexample to an argument with these premises and this conclusion. In fact, we will see that these derivations have the property of *soundness*, which expresses exactly that.

- LEARN BASIC META-THEORETICAL RESULTS: Finally, you will get a brief introduction to some of the most important theoretical results which have wide ranging implications: the soundness property mentioned above, and then the completeness theorems for TFL and FOL arguments.

- Lastly, you should begin to see why and how mathematics and computer science make use of formal languages of just the sort (FOL) to which you are exposed here.

Advice Many students standardly find the second half of the course (on FOL) much more difficult and challenging than the first. This is completely normal. However, it means that finding the initial stages easy is *not* a sign that you will find the whole course easy. In addition, understanding the material is also by its nature cumulative; one cannot neglect the course for a few weeks, and then expect to understand new material without the background understanding of the old. (In a course of 19th/20th c. European History, one might get bored with all the intricacies of the Russian Empire's interference in the Balkans, and move straight to the origins of the First World War. You could not do a similar thing here.) If you find yourself behind, you must try to catch up in order, and not to skip. Lastly, don't worry about making mistakes, but learn to learn from your mistakes, and ask why you made them: ask yourself, ask your peers, ask the Textbook, ask the Slides, ask us.

Course Material, Organisation, and Lectures

1. The basis of the course will be the Open Source textbook *For All x: Calgary* (Richard Zach et al.), found at `http://forallx.openlogicproject.org/`, available also on *myCourses*. Make sure you get the 'Fall 2023' version (last edited in August 2024). This is the edition posted on *myCourses, Content/Textbook*.

2. There are other version of this text available at that web site, e.g., an HTML version, a version designed for those with dyslexia, a B/W version, a version for printing, and more. The Fall 2023 Edition of the book can be purchased on Amazon at a modest price. On-screen reading has many advantages, but reading the text in a book has unique advantages, too.

3. Whatever medium you choose, there is no substitute for close reading of the book, and for doing the bulk of the many exercises it contains. Some of these will be routine, and some more challenging, not just asking you to reiterate what you have learnt, but to use this in ways which at first sight might not be entirely clear. However, the main aim is to understand *why* the mechanisms presented work, and not simply to be able to apply them. Correspondingly, some of the questions on the tests and assignments will be *theoretical* in nature.

    - Logic is one of those subjects where proficiency is only acquired through practice. This means that doing the exercises in the book regularly is *essential*. Moreover, if you don't do these exercises regularly, you will find it hard to complete the quizzes, problems and exams on which your mark ultimately depends.

- We will occasionally upload sets of practice exercises to the *Carnap* system to be described below. This will enable you to test out your answers on that system (which in the case of practice exercises offers some level of help), but also serve to familiarise you to the *Carnap* system itself. This system will be an essential component of marked assignments. *It would be a serious mistake to go into a timed* Carnap *assignment without being familiar with the Carnap system*.
- Note that we will not use all chapters in this textbook, though some additional material might be covered in Lectures.

4. The exposition in the lectures will broadly (but not exactly) follow that of the textbook. If anything, the lectures will concentrate more on the *theoretical* side of the material rather than on examples; these will be dealt with more in the exercises, and in your work with the TAs.

5. A *Provisional Schedule* for the course, detailing the Chapters to be read each week will be published on *myCourses* after the beginning of the semester. As its title makes clear, this is a *provisional* schedule, an *informal* guide to the order in which matters are dealt with, and to the reading (and implicitly exercises) you ought be doing at a given stage of the course. The actual course might deviate from the idealised plan outlined in the Schedule.

6. The lectures (1 hour on Monday, 1 hour on Wednesday) will be delivered with weekly Slides, which will then subsequently be posted on *myCourses*. This means you will have the Slides at least *twice*, once as they are presented in lectures, and then in reading through them yourself later. Not everything on the Slides will be discussed in the lectures, just as not everything in the Textbook will be covered, not even from those Chapters we concentrate on. Nevertheless, we will feel free to base questions on relevant and germane material not discussed, but which is covered in readings assigned or in the Slides.

7. There may well be Handouts covering some of the more difficult topics; these you will be informed about if and when they are issued.

Conferences

As well as the lectures, you will also have weekly meetings (Conferences) with TAs. The (many) Conferences will be set at various times during the latter half of the teaching week, but there will always be some set for the Friday 13.35–14.25 slot, which should be free for all students in this course. At some point you will have to register with a TA and a specific conference on Minerva, once registration is open. The main task of the TA will be to go over some exercises, and to help you with particular difficulties you have. It will be important to bring concrete problems you've had to the Conferences.

Conferences normally start in the second full week, and an announcement will be made when they are ready for registration and to start. *Note that conference size is limited; please sign-up promptly to get the time most convenient to you.*

As implied above, formal logic is very much a subject where practice is essential, and where the exercises really instil familiarity with the material. Therefore, regular contact with the TA is very important. All the TAs will standardly have Office Hours during the week, as will I, and you can go to whichever OH suits your schedule, more than one if you wish. The arrangements for these will be worked out and posted soon. PLEASE MAKE USE OF THEM.

myCourses

McGill's *myCourses* system will be crucial for the course. This is where the Description, Schedule, Slides, Explanation of the *Carnap* system, demonstration videos, Handouts etc. can be found, it's where important components of the Assignments will appear or where they have to be uploaded, and it's where important Announcements will appear noting details about access to various resources, forthcoming deadlines,

changes, exercises, Assignments. Also some parts of the Assignments will actually run exclusively through the *myCourse* site itself. (See below.)

Please note that the *myCourses* system uses *only* your official McGill e-mail address, so this (as well as *myCourses*) should be checked regularly and routinely. The use of *Carnap*, however, requires a Google registration, so you will have to use *both* addresses. More below. $\mathcal{NB}$

Marking and Assessment There will be 2 Assignments and a Mid-Term Exam for marking, all to be taken on-line, and a Formal Final Exam which will be scheduled in the examination period. The Assignments will be worth 15% each, the Mid-Term 30%, the Final Exam 40%. Be aware that the Final Exam might have questions on any aspect of the course. Some of the questions will be similar to questions you will have encountered in the Exercises, the Assignments, and the Mid-Term, but there will also be other questions about definitions and important logical concepts. $\mathcal{NB}$

NOTE THAT A PASSING MARK ON THIS FINAL WILL BE NECESSARY FOR PASSING THE COURSE. $\mathcal{NB}$

Each of the Assignments/Mid-Term will standardly be released on-line on Fridays at 17h00 EDT, and due the following Monday at 23h59 EDT (so roughly 3.5 days). All three will be divided into two components. The first component (worth 10% or 20% for the Mid-Term) will be a timed test completed on the Carnap system. The assignments should take about 1.5 hours to complete, but you will be given more time than this to do so. The remaining 5% (10% in the case of the Mid-Term) will be earned by answering a relatively small number of questions on the *myCourses* 'Quiz' platform. Again, you will be given a generous amount of time within the $\approx$ 3.5 days to complete them.

POLICY FOR EXTENSIONS AND LATE WORK: Extensions will be granted only in exceptional circumstances (illness (incl. mental health), death of a family member, political instability, major power outages, that sort of thing). Note: only the lecturer can grant an extension; *please do not ask the TAs*. Late work without a granted extension will be penalized at the rate of a full letter grade (12.5%) per day overdue. Thus, an assignment judged to be worth a B+ (or around 77%) but late one day will be assigned C+ (or around 64.5%), late two days D (50%), and so on. $\mathcal{NB}$

Should you anticipate requiring an extension, please notify me as soon as possible.

The Carnap System for Exercises and Assignments For both Exercises and Assignments, we will make use of the on-line system to be found at the website `http://carnap.io`. To be able to use this, you have to register with it *using a Google gmail address*. Once registered, you will then be able to choose a course (there are many), and you will choose the appropriate McGill one, namely 'Phil 210, McGill University, Fall 2023'. (The enrollment link is: `https://carnap.io/enroll/Phil%20210,%20McGill%20University,%20Fall%202023`.) After you've done this, there will appear *eventually* in your Home Page in *Carnap.io* a list of sets of exercise practice questions (each set characteristically marked with 'Week *n*' or something similar) and then later files marked as Assignments, which will have starting and due dates. At the moment, the course is empty, but as it progresses, more and more things will appear there, and it will be clear from the title whether what appears is for practice or for marking. $\mathcal{NB}$

There will be a special section in *myCourses* about registering for and using the Carnap system.

The Carnap interface is very easy to use; above all, it accepts ordinary keystrokes instead of special symbols, and will produce for you the special symbols. (E.g., for the special symbol '$\wedge$' which we use, Carnap will accept '&', 'and' and more, but will display '$\wedge$' back to you in its output. Similarly later for the symbol '$\forall x$', where it will accept ordinary things like 'Ax' or 'All x'. Don't worry, the files posted exercises will make it clear what's accepted, along with other relevant guidance.) This means that we can use the system to write sentences easily in the two artificial languages we study, to assess various properties of these sentences, and then to produce Fitch proofs.

I will also post some instructional videos on how to use the Carnap.io system. These begin at the beginning, and then gradually introduce you to the more complicated things we do with the system. The videos are not essential, but will be of some assistance to those who find the Carnap system difficult at first. The videos stem from the dark days of COVID, and vary in length and explicitness. (Most people will able to play them on 1.5 or 2.0 speed to give a faster overview.) The advantage of them is that they show on the screen my operating with the Carnap system just as it's presented to you, and then you can pause the video and try similar things yourself. The investment of time to grasp the basic and then more sophisticated details is well worth it. $\mathcal{NB}$

McGill Policies

1. *McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures. (See* `www.mcgill.ca/integrity` *for more information.)* $\mathcal{NB}$

2. *In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.*

3. *In accord with McGill University's Charter of Students' Rights, students in this course have the right, without seeking permission, to submit in English or in French any written work that is to be graded.*

4. *As instructors of this course, the Lecturer and TAs endeavor to provide an inclusive learning environment. If you experience barriers to learning in this course, do not hesitate to discuss them with us or with Student Affairs or the* Office for Students with Disabilities, `https://www.mcgill.ca/osd`, *514-398-6009.*

5. *McGill University is on land which is the traditional and unceded territory of the Kanien'keha:ka (Mohawk), a place which has long served as a site of meeting and exchange amongst nations.*