Phil 310 *Intermediate Logic* Winter 2018

Tuesday, Thursday: 11.35–12.55, Leacock 14.

Course Page: *MyCourses*

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Summary. In a first course on logic, the emphasis is on working *within* the standard logical system; in this course, however, we concentrate much more on proving theorems *about* the standard logical systems. Because of this, the course is quite different in character from an introductory logic course: above all, it is more mathematical in nature. Because of this, *mathematical aptitude* (although not necessarily any special mathematical *knowledge*) is very important. This is meant to be a **WARNING**, for many students (by no means all, of course!) are misled, having found introductory logic easy.

By way of warm-up, we will begin with a brief introduction to types of proofs, and then some basic set theory. This will be followed by a brief presentation of *propositional logic*, focusing on the Completeness Theorem, the first major result not proved in the introductory course. After this we shall concentrate our studies on *classical first-order predicate logic*. In particular our focus will revolve around two major results: (1) the *Completeness Theorem for first-order logic*, and then (2) *Gödel's First Incompleteness Theorem*.

Surrounding (1), we shall present Henkin's proof of the Completeness Theorem itself, the *Compactness Theorem*, which can be invoked to show the existence of non-standard models for arithmetic (*Skolem's Theorem*), i.e., the non-categoricity of the first-order axioms for arithmetic. This will provide an important bridge to the material concerning (2). Surrounding (2), we will prove Gödel's Theorem itself, and leading up to this, we will present important elements of recursive function theory, *Gödel numbering* and *representability*; we will also give accounts of *Church's Theorem* on the undecidability of first-order logic, *Tarski's Theorem* on the undefinability of truth, and finish with Gödel's *Second Incompleteness Theorem* and an account of the closely related *Löb Theorem*. Many of the results grouped together under (2) concern the overarching question of the ability of logical systems to represent mathematics *adequately*, even the basic theory of natural numbers.

By studying these results (some of the most important theorems of twentieth-century logic) we will learn a great deal about the power and the limitations of first-order logic. These technical results have far-reaching philosophical implications (e.g., computational theory of mind, the theory of truth, the nature of mathematical and scientific theories, the role of proof and provability, Hilbert's programme), not just for the study of the philosophy of mathematics, but for philosophy generally.

The material is very much cumulative, and the results emerge slowly. Hence, it's important to keep up, and to be patient. Believe us, it's worth it! And remember, what we're doing is really proving things in an *informal* way about *formal* proof systems, even though it helps to be familiar with doing formal proofs (following rules of proof) and with the semantics of first-order languages.

Prerequisites. Introduction to Deductive Logic (PHIL 210), COMP 230, or equivalents. Not open to students who have taken MATH-498.

Textbook. The lectures will follow closely the development in the second half (Chs. 7–10) of

- Moshé Machover: *Set Theory, Logic and Their Limitations* (Cambridge, 1996).

The book will be available at The Word Bookstore, 469 Milton Street (5 mins. from the University Street Gates). This text is **essential**. [NB. The Word does not accept credit cards, only cash or cheque.]

Requirements & grading. Students will be required to attend and participate in class, do the assigned readings, complete weekly homeworks, and take a final exam. The final grade depends on homeworks (70%), final exam (25%), and participation in the course (5%). Failure to hand in the homeworks in time will result in the loss of marks.

Reading and Handouts I will issue Handouts regularly; these will be made available through *MyCourses*, so you should keep a steady eye on this. The Homework will also be issued in the same way. I may also post extra readings periodically. This will include some initial background material which I recommend reading *before* the course proper begins. The most interesting of these readings is an article 'Gödel's proof' by Ernest Nagel and James Newman, published in 1957 in *Scientific American*. This presents the very important background which culminates in Gödel's Incompleteness Theorem, and has an informal presentation of the result itself. (The article was later expanded into a small book, which you can find in the library.) Gödel's Theorem is the final important goal of the 310 course, so understanding the background and having some idea of the way the result is proved cannot but be beneficial. There will also be some readings from Frege, the originator of modern logic, and also some material on the nature of logic itself. These readings will be placed on the *MyCourses* site in the 'Introductory Readings' folder.

Some of the material we cover in Phil 310 is dealt with in less detail in Chs. 16–19 of the book *Language, Proof and Logic* (by Barwise, Etchemendy *et al.*) which is used for the introductory logic course, Phil 210, although this material is beyond the scope of a standard introduction to logic. However, it is a good idea to look at these sections *now* to familiarise yourselves with some of the ideas. For those of you who do not possess the book, I have put the relevant chapters on the *MyCourses* site, again in the 'Introductory Readings' folder.

Three other good books on the material covered here are: (a) Hubert Enderton: *A Mathematical Introduction to Logic;* (b) Elliot Mendelson: *Introduction to Mathematical Logic*; and (c) George Boolos, John Burgess and Richard Jeffrey: *Computability and Logic* (4th/5th edition, Cambridge University Press). A very nice book (which, however, doesn't go as far as we will) is Dirk van Dalen: *Logic and Structure* (3rd edition, Springer-Verlag). The best book in French is S. C. Kleene, *Logique mathématique* (Paris, Armand Colin, 1971, a translation of Kleene's *Mathematical Logic*).

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1. *McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see* `www.mcgill.ca/integrity` *for more information).* NB

2. *In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.*

3. *Students have the right to submit work in French..*